UNIT 1

Area and Surface Area

Geometry is the mathematics of space and all the objects in it, which come in various shapes and sizes, and even dimensions. You know the names of many special two-dimensional and three-dimensional figures, and have worked with the area of very basic shapes before. But now it is time to cover anything and everything, literally.

Essential Questions

- What does it mean when you say two shapes have the same area?
- How is surface area different from volume?
- *(By the way, what do you get when you stitch together* 12 *pentagons and* 20 *hexagons?)*

SUB-UNIT

1 Area of Special Polygons

Narrative: Discover how something can never be greater than the sum of its parts.

You'll learn . . .

- about decomposing shapes.
- formulas for the areas of triangles and parallelograms.

SUB-UNIT

2 Nets and Surface Area

Narrative: From cardboard boxes to suspended tents, areas folded in three dimensions have got you covered!

You'll learn . . .

- how nets can be used to compute surface area.
- to represent repeated multiplication with exponents.

A cube has N unique nets, two of which are shown below. If you add up the digits of N and square the result, what number do you get?

The Tangram

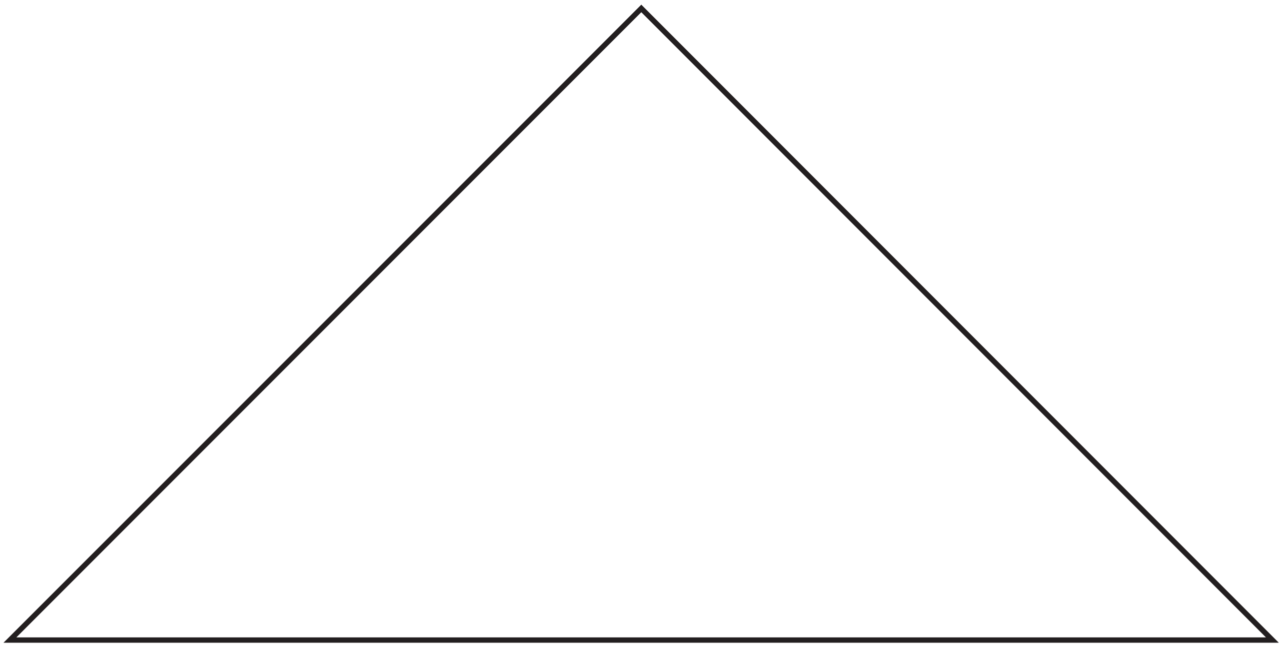
Let's discover the tangram.

Warm-up Working Together

Work with your partner to create the figure shown here using all seven of your tangram pieces.

As you work, pay attention to and be prepared to share your responses to the following with the class:

- What happened next when you and your partner "got stuck?"
- How were moments of disagreement resolved?
- How were you and your partner able to use your time productively?



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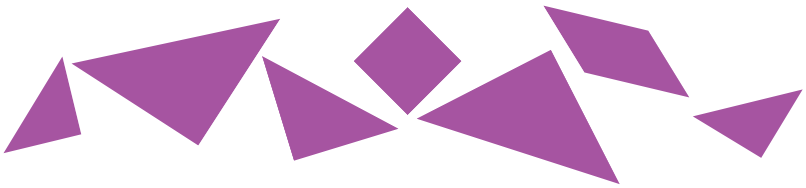
Activity 1 The Tangram Legend

Use your seven tangram pieces to recreate the images from *The Tangram Legend.*

Many centuries ago, a king ordered a window be created for his palace in the shape of a perfect square. A sage was sent out on the arduous journey to collect the glass from an artisan who lived on the opposite side of the kingdom.

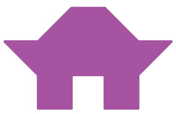
The long route involved navigating a vast array of landscapes — fields, forests, deserts, and rivers. Nearing her final destination, the sage climbed the rockiest peak of the final mountain range, and the palace came back into view.

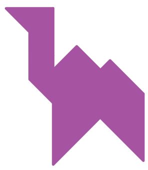
Overjoyed by her imminent arrival and a sure-to-be pleased king awaiting, the sage took a hasty step and tripped. The tumble down the mountain broke the precious glass. But intriguingly, it was not shattered — rather, seven geometric shapes, each equally impressive, were formed.



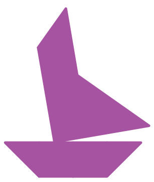
The heart-broken sage came before the king and recounted her treacherous journey. As she spoke, she skillfully moved the shapes around and formed many images to recreate the journey.

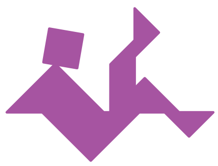
The *home* of the artisan. A *camel* used to cross the desert.



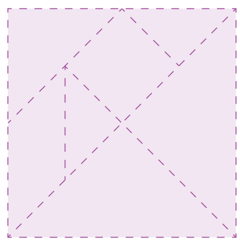


A *boat* for sailing across the river. The infamous *mountain range* and the apologetic *falling*.





The king was so fascinated by the multitude of geometric images that could be created from the pieces that he had the shapes recreated out of wood. Thus, the *tangram square* was invented.



Activity 2 Tangram Paradoxes

A *paradox* is a statement that seems like it cannot be true at first, but after investigating it or thinking about it in a different way, it seems like it *could* be true.

Work with your partner to recreate both figures in each of these two paradoxes and explain what is happening that allows each to be possible.

1. Figures A and B are both squares that can be created using all seven tangram pieces, but one has a hole in the middle.

Figure A

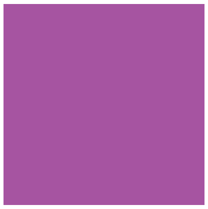
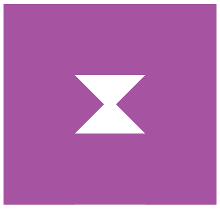


Figure B



2. Figures C and D are both side views of a person that can be created using all seven tangram pieces, but one has feet!

Figure C

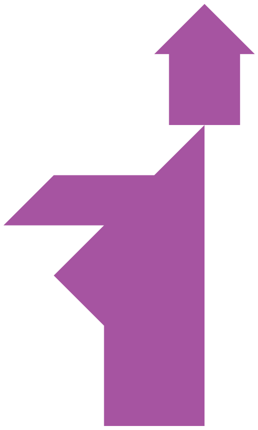
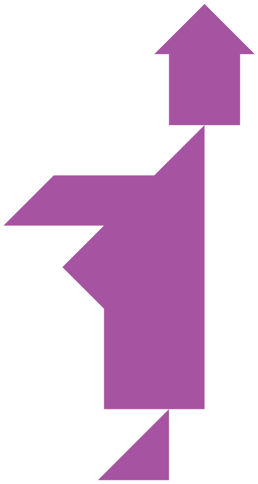


Figure D



STOP

Unit 1 Area and Surface Area

A Place for Space

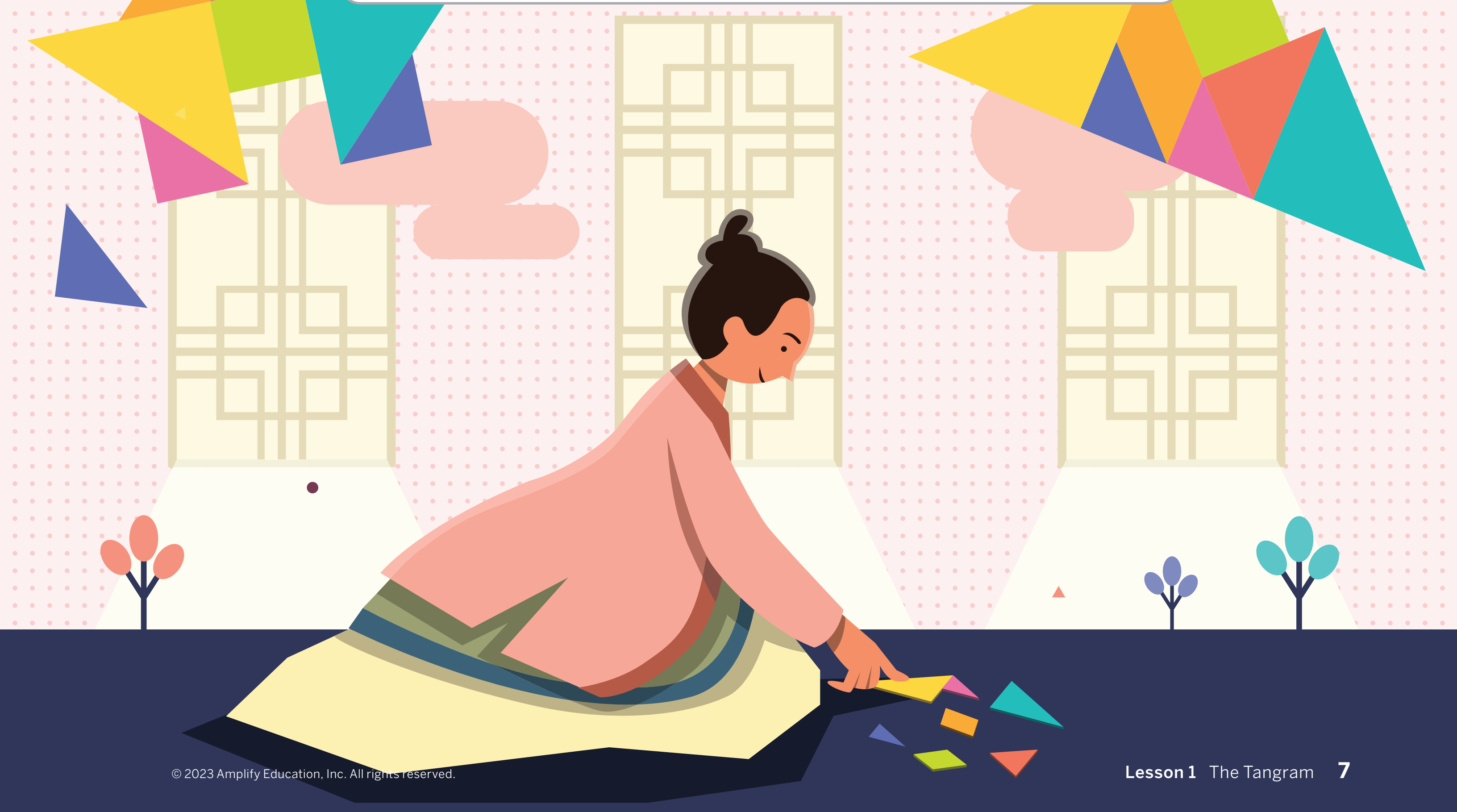
At first glance, a tangram might look like a simple toy: a flat square, made up of seven smaller shapes, called “tans.” But using only these seven shapes, you transformed the square into a house, a mountain, a camel, and a boat. No one knows the true history of the tangram, but the legend of the sage who shattered a precious glass pane shows how entire stories can be woven together with just those seven shapes.

It was the artist Michelangelo who said, “Every block of stone has a statue inside it.” He could have just as easily been talking about tangrams — or even *math students*, for that matter.

As you start the year, think about the humble square. Think about all the possibilities tucked inside it. And just as there is a statue inside every block of stone, there is a mathematician within every student. It’s right there, just waiting to be brought to the surface.

With an open mind, some elbow grease, and a little imagination, we’ll get there together.

Let’s see how . . .



Practice

Name: ______ Date: ______ Period: ______

1. Identify two of your strengths as a math student and two areas in which you would like to grow or improve as a math student.

2. What motivates you to do your best?

3. What does it mean to be accountable to yourself? To your peers?

Name: .... Date: .... Period: ....

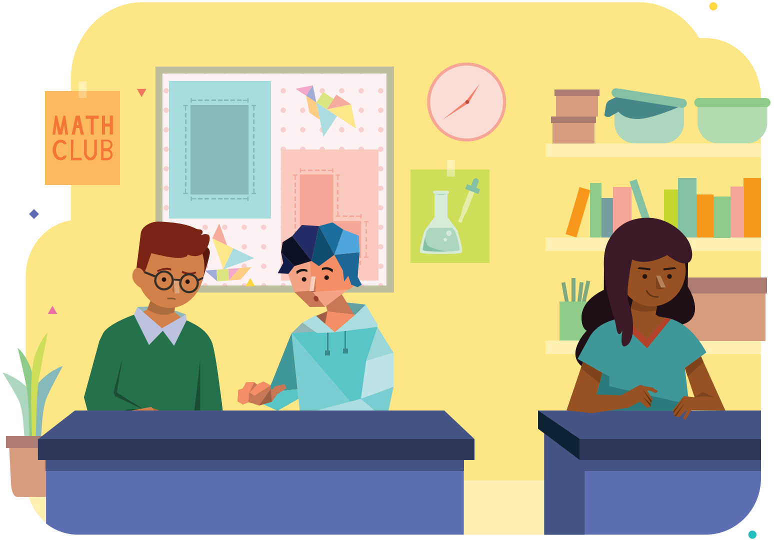
Practice

4. How can you promote positive and effective communication with others?

5. What are some qualities that you would like your peers to display when collaborating and working together?

6. Using the tangrams from class, make a quadrilateral. Sketch your quadrilateral.

Exploring the Tangram



Let's make patterns using tangram pieces.

Warm-up How Many Squares?

Using four or more of the pieces from a tangram set, how many different ways can you build a square of any size? Record the tangram pieces used for each square that your group builds, and then sketch the square.

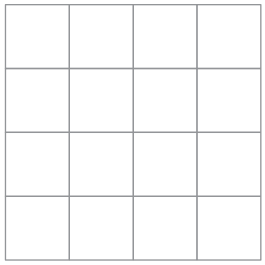
Compare and Connect: After completing the Warm-up, share with a partner what is different and what is similar among the ways the squares have been composed.

Name: .... Date: .... Period: ....

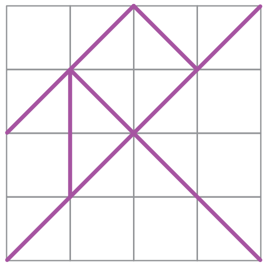
Activity 1 Making a Tangram Set

You will be given a square piece of paper. Follow the steps to create your own set of tangram pieces from that square.

1. Fold the square piece of paper in half horizontally, and then fold it in half again vertically. Repeat these two types of folds one more time. When you unfold the paper, the folds create sixteen equal-sized squares.



2. Draw lines on your square as shown here, and then cut your paper along the lines.



3. You will now have a set of the seven standard tangram pieces:
   - one small square
   - two small triangles
   - one medium triangle
   - two large triangles
   - one parallelogram

Are you ready for more?

Each of these figures represents a different paradox. They can all be solved using all seven tangram pieces. But they can also all be solved using only six tangram pieces. Try to solve one (or more) of these tangram puzzles, first using all seven pieces, and then again using only six pieces.

Figure A

Figure B

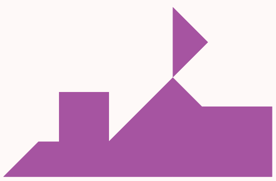


Figure C

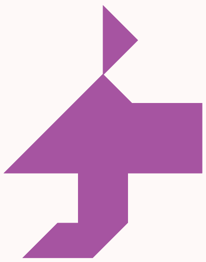
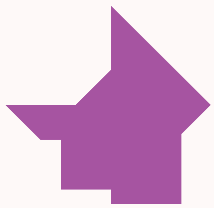


Figure D



Activity 2 Creating Your Own Tangram Puzzle

The classic rules of tangram puzzles are:

- **All seven tangram pieces must be used in the puzzle.**
- **All pieces must lie flat.**
- **Each piece must touch at least one other piece.**
- **No pieces can overlap.**

Create a puzzle using the tangram pieces you created in Activity 1. Draw an outline of your puzzle and its pieces here. Then glue the pieces on a separate sheet of paper and color them.

STOP

Unit 1 Area and Surface Area

A Place for Space, continued

Originating in China during the Song dynasty, tangram puzzles spread to the U.S. and Europe in the early 1800s. They fascinated figures as wide-ranging as Napoleon Bonaparte, Edgar Allen Poe, John Quincy Adams, and Lewis Carroll.

The beauty of tangrams is that they take something simple—a square—and turn it into something deeply complex. And just as we rearrange tans to compose a shape, we also have to rearrange the way we think to arrive at our solutions.

In some ways, your math class is like a tangram. It's a whole unit on its own, but it's also made up of smaller individual pieces: you, your classmates, and your teacher. Each of you brings something different and valuable to the room. You bring your ideas, your curiosity, your creativity, your perseverance, your stories. These are the elements you need for a math class to work.

Whether it's the tans in a tangram, or a student in a class, understanding how things are put together starts with understanding the individual pieces.

Welcome to Unit 1.

Practice

Name: .... Date: .... Period: ....

1. Think about this upcoming year in your math class.

a Describe one goal you have for this year in math class.

b What is:

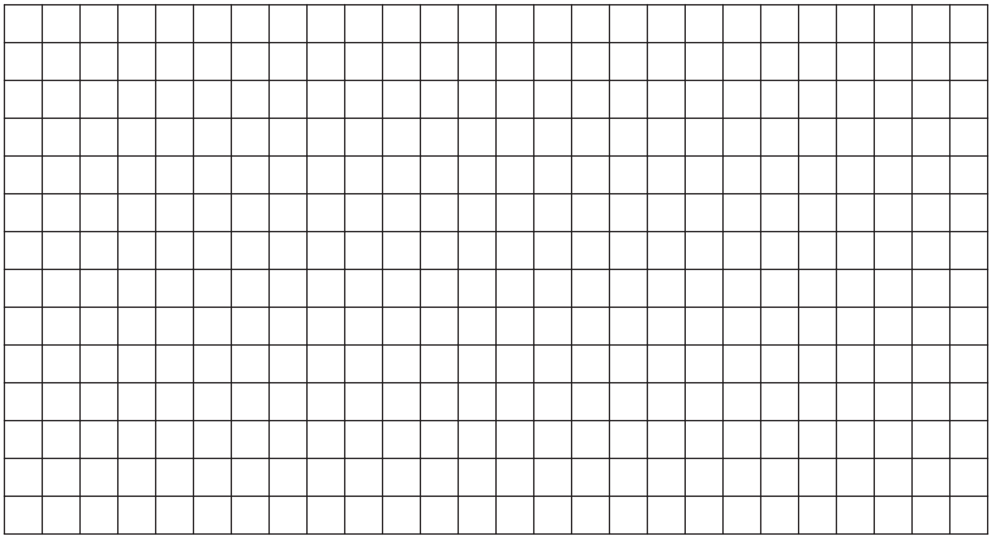
- one way you can help yourself reach your goal?
- one way your teacher can help you reach your goal?
- one way your peers can help you reach your goal?

2. What is most important to you about the cycle of giving and receiving constructive feedback?

3. What are the seven pieces that make up a tangram?

Practice

4. Each square in this grid has an area of 1 square unit. Draw three different quadrilaterals that each have an area of 12 square units.



5. For each statement about parallelograms, determine if it is *always* true, *sometimes* true, or *never* true.

a) Opposite sides are parallel.

a) All angles are right angles.

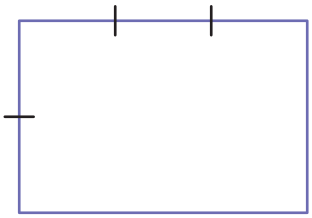
c) Opposite angles are equal.

d) Opposite sides are different lengths.

e) A parallelogram is also a rectangle.

f) A rectangle is also a parallelogram.

6. The side lengths of the rectangle shown are 3 cm and 2 cm. What is the area of the rectangle?



My Notes:

Can a sum ever *really* be greater than its parts?

Nope. At least, not when it comes to geometry.

Most anything you can take apart, you can also put back together. But putting things together is the hard part!

You got a taste of this challenge with the tangrams you just saw. And if you've ever tried to assemble furniture, you know that things may not end up looking how they're supposed to, even when you use all the pieces.

So, while two different figures can be made up of exactly the same parts and take up the same amount of space, they can *still* look completely different.

When we're talking about two-dimensional figures, the amount of space a shape takes up is called its *area*.

Different shapes have their own relationships between their area and their dimensions — that is, their lengths. And if you know the area of one shape, you can figure out the area of any shape that can be made from it. As you'll see, when it comes to polygons, it always comes back to the humble triangle.

Unit 1 | Lesson 3

Tiling the Plane

Let's look at tiling patterns and think about area.

Warm-up Exploring Your Geometry Toolkit

Take turns choosing tools from your geometry toolkit and discuss with your partner how you think each tool might be used in this geometry unit.

Record the names of the tools here, as you discuss them.

Log in to Amplify Math to complete this lesson online.

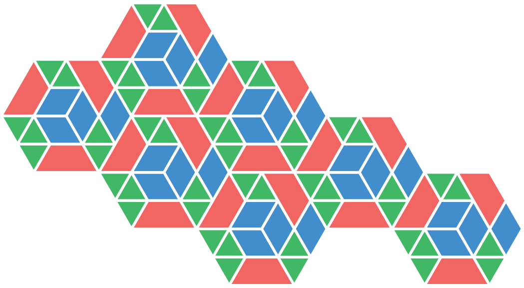
Name: .... Date: .... Period: ....

Activity 1 Tiling the Plane

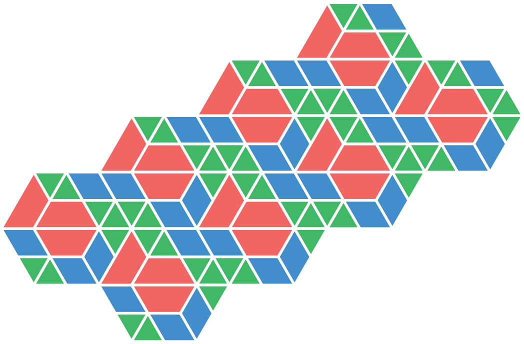
Filling spaces with tiles of different shapes and colors can be both decorative and beautiful. It is also very mathematical; mathematicians, such as Laura Escobar, have studied the connections between certain patterns of rhombus-shaped tiles and how letters can be ordered in different ways.

For now, you will be assigned either Pattern A or Pattern B. Determine which shape — rhombus, trapezoid, or triangle — covers more of the plane in your pattern. Be prepared to explain your thinking.

Pattern A



Pattern B



Collect and Display:
Your teacher will circulate and collect key terms and phrases to add to a class display as your group discusses the patterns. Refer to this display during future discussions.

Featured Mathematician

Laura Escobar

Hailing from Bogotá, Colombia, Laura Escobar is a professor of mathematics at Washington University in St. Louis, Missouri. Her research lies at the intersection of algebra, geometry, and combinatorics (the mathematics of how things are arranged, and how many ways there are to arrange them). In 2016, she was the lead author on a paper that explored connections between special tilings of rhombuses, combinations of letters, and what are known as “Bott-Samelson varieties.”

STOP

Summary

In today's lesson . . .

You looked at copies of the same two-dimensional shapes being placed together in different ways, but always such that there were no gaps or overlaps. In thinking about which shapes make up more of a pattern or cover more of a region, you reasoned about *area*. Particularly, you revisited the idea of what it means for two shapes to have the same area.

This is just the start of the work you will do this year. You will use mathematics and the tools of mathematicians to answer questions, as well as ask and answer your *own* questions. You will continue this work and discover more flexible and efficient uses for the tools in your geometry toolkit, as you explore more about the concept of area in this unit.

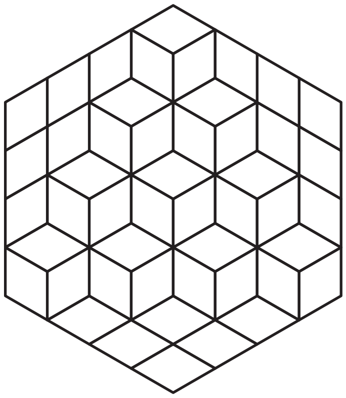
Just as important as understanding mathematical ideas for yourself, this lesson presented the first of many opportunities to practice speaking like a mathematician, by sharing your understanding and thinking with others. Also just like mathematicians, you worked together with partners and groups of classmates, as well as with your teacher, to help you arrive at your own understanding, while also considering the perspectives of others.

Reflect:

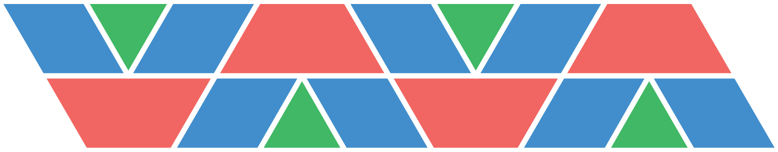
Name: ________________ Date: ________ Period: ________

Practice

1. What tool(s) could you use to help you visualize the shapes in this pattern?



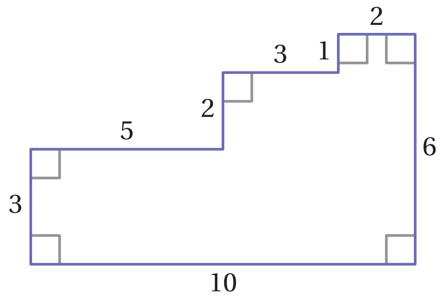
2. Using only triangles, how many triangles would be needed to cover this pattern? Show or explain your thinking.

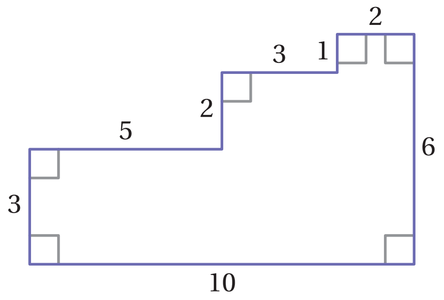


Practice

Name: .... Date: .... Period: ....

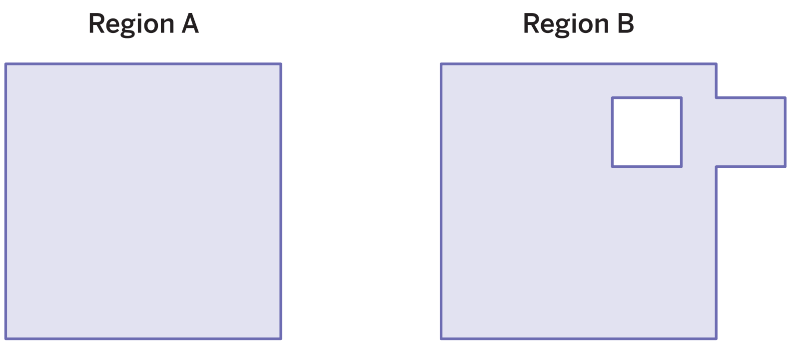
3. Here are two copies of the same shape. Show two different ways for determining the area of the shape. (Note. You don not need to calculate the actual area.) All angles are right angles.





4. Which shape has a larger area: a rectangle that is 7 in. by $\frac{3}{4}$ in., or a square that has a side length of $2\frac{1}{2}$ in.? Show or explain your thinking.

5. Which shaded region covers more area? Show or explain your thinking.



Name: ______ Date: ______ Period: ______

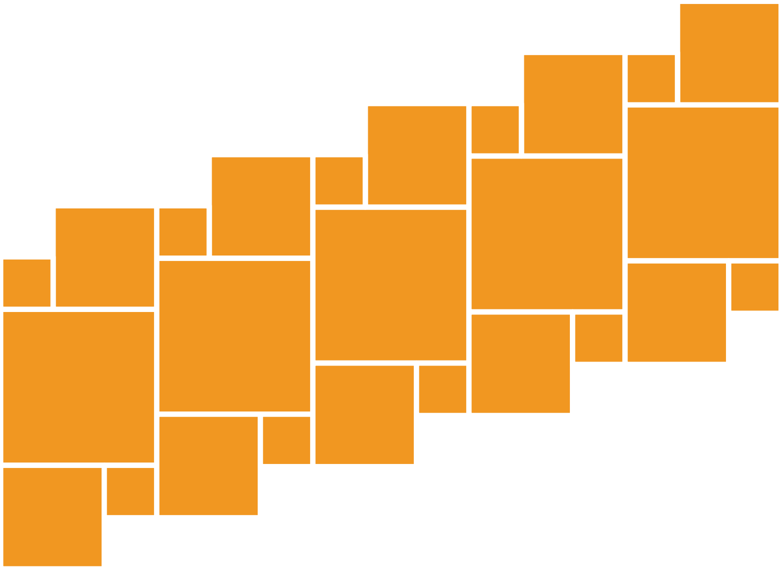
Unit 1 | Lesson 4

Composing and Rearranging to Determine Area

Let's create shapes and determine their areas.

Warm-up Comparing Regions

Which type of square — large, medium, or small — covers more area in this pattern? Consider using the tools in your geometry toolkit to help your thinking.



Reflect: Did you use any tools from your geometry toolkit to help you? Why or why not?

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Activity 1 Composing and Rearranging Shapes

You will be given a square and several different-sized triangles. The area of the square is 1 square unit.

Part 1: Composing

1. What is the area of two small triangles when they are placed together? Be prepared to explain your thinking.

2. Use two or more pieces to create a new shape — that is not a square — with an area of 1 square unit.

   a. Draw your shape.

   b. What is the area of each piece?

3. Use as many of the pieces as you need to create a shape with an area of 2 square units.

   a. Draw your shape.

   b. What is the area of each *different* piece in your shape?

Name: ______ Date: ______ Period: ______

Activity 1 Composing and Rearranging Shapes (continued)

Part 2: Rearranging

Use exactly the same pieces from Problem 3 to create a *different* shape than you created in Problem 3.

4. Draw your new shape.

5. What is the area of this new shape?

Part 3: Composing and Rearranging

Starting with the same pieces from Part 2, use additional pieces to add to your shape to create a new shape, now with an area of 4 square units.

6. Draw your shape.

7. What is the area of each *different* piece in your shape?

Are you ready for more?

Show how you can use all of your pieces to compose a single large square. What is the area of this large square?

STOP

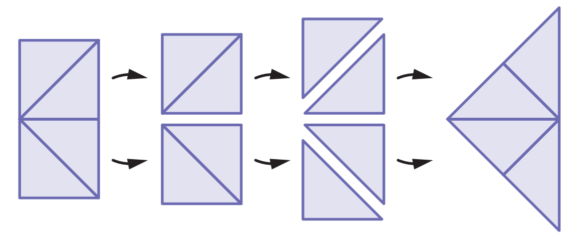
Summary

In today's lesson . . .

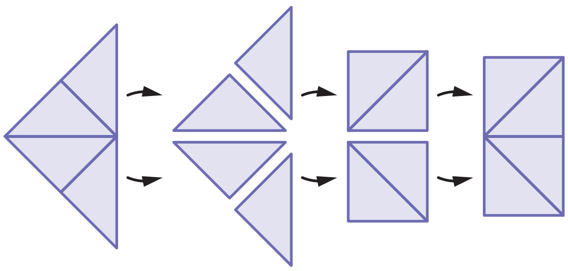
You used two important principles of area:

1. If two shapes can be placed one on top of the other, so that they match up exactly, with no gaps or overlaps, then they have the same *area*.
2. You can ***decompose*** (break it into pieces) a shape and ***rearrange*** (move them around) the pieces to form a new shape. The area of the original shape and the area of the new shape are the same.

Here are two illustrations of the second principle.



The rectangle can be decomposed into two squares, and then each square can be decomposed into two triangles. These four triangles can be rearranged to form the large triangle. So, the large triangle has the same area as the rectangle.



The large triangle can be decomposed into four smaller triangles. These four triangles can be rearranged to form two squares, which can be ***composed*** (placed together) to form the rectangle. So, the rectangle has the same area as the large triangle.

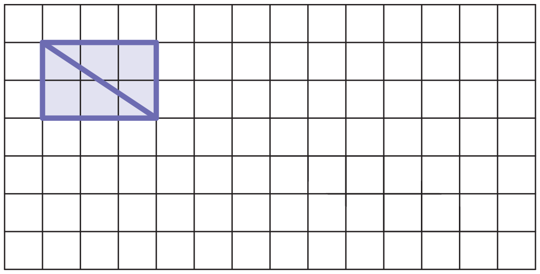
If each square in these illustrations has an area of 1 square unit, then the area of a small triangle is $\frac{1}{2}$ square unit, and the area of the large triangle is 2 square units.

Reflect:

Name: ______________________ Date: __________ Period: __________

Practice

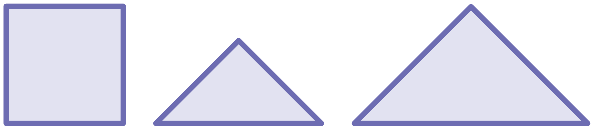
1. The figure shows a rectangle on a grid that is composed of two triangles.



a. Rearrange the two triangles to make a *different* shape.

b. How does the area of this new shape compare to the area of the original rectangle? Explain your thinking.

2. The area of the square is 1 square unit. Two copies of the smaller triangle can be arranged to form either of the other two shapes — the square or the larger triangle. Which figures have an area of $1\frac{1}{2}$ square units? Select *all* that apply.



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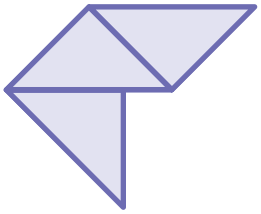
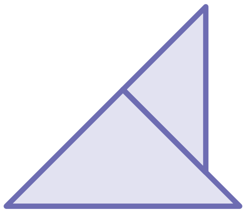


C.



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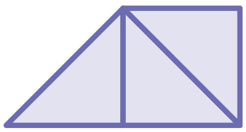

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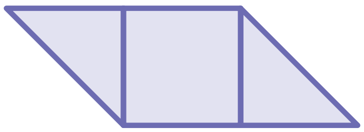
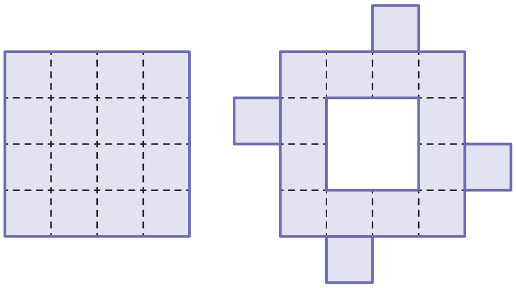


Practice

Name: ____________________ Date: __________ Period: __________

3. Priya decomposed a square into 16 identical smaller squares. She then cut out 4 of the small squares and placed them around the outside of the original square to make a new shape, as shown. Which of these statements accurately describes how the area of her new shape compares to the area of the original square?



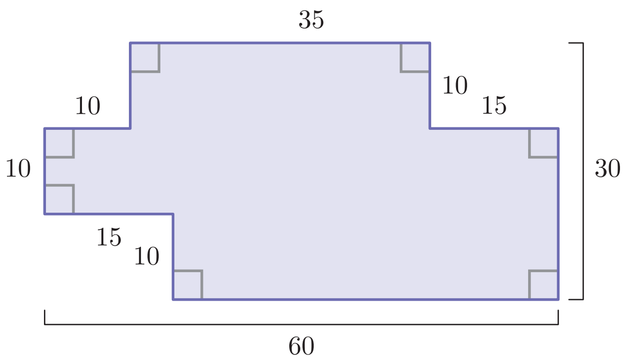
A. The area of the new shape is greater.

B. The two shapes have the same area.

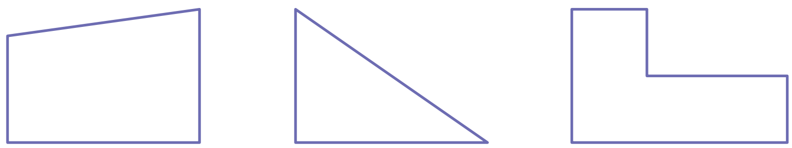
C. The area of the original square is greater.

D. It is not possible to tell because neither the side length nor the area of the original square is known.

4. Tyler studied the figure shown and said, “I cannot determine the area because there are many different measurements, instead of just one length and one width that could be multiplied together.” Explain why Tyler’s statement is incorrect.



5. For each polygon, circle any angles that appear to be right angles.



Name: ______________________ Date: __________ Period: ________

Unit 1 | Lesson 5

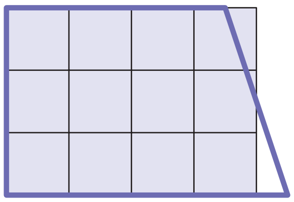
Reasoning to Determine Area

Let's use different strategies to determine the area of a shape.

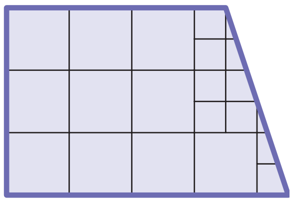
Warm-up What Is Area?

1. Which representation would you use to determine the area of the trapezoid? Be prepared to justify your choice.

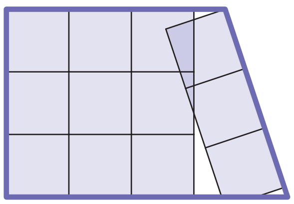
A.



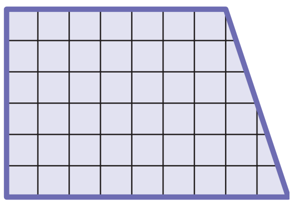
B.



C.



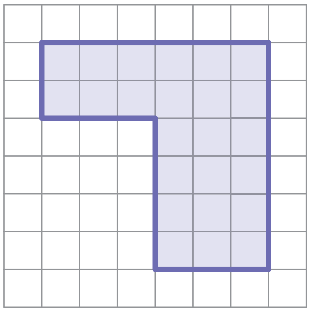
D.

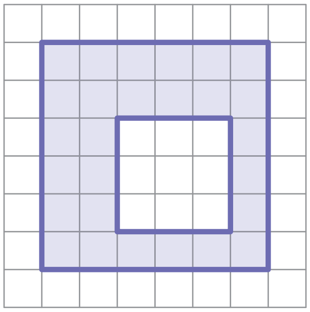


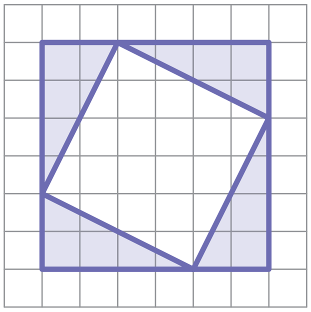
2. After the discussion, record your class definition of *area* here:

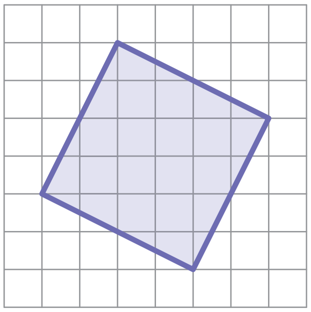
Activity 1 On the Grid

Each small square in these grids has an area of 1 square unit. Show or explain how to determine the total area, in square units, of each of the shaded regions without counting every square.

1. 

2. 

3. 

4. 

Critique and Correct:
Your teacher will provide you with a sample response for Problem 3. Work with your partner to determine what the author meant and whether or not the response could be improved.

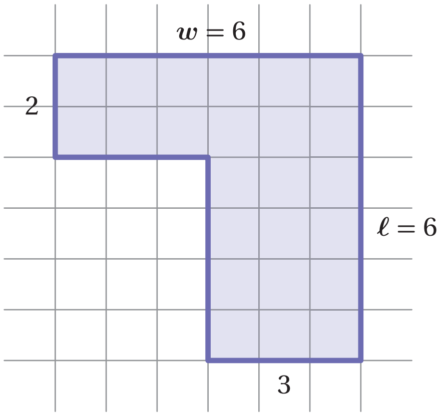
Name: ______ Date: ______ Period: ______

Activity 2 Off the Grid

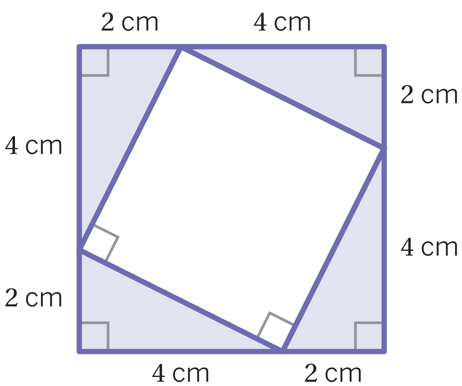
Plan ahead: How will you organize your thoughts so that you can clearly communicate them to others?

Determine the total area of all of the shaded regions in each figure. Explain or show your thinking.

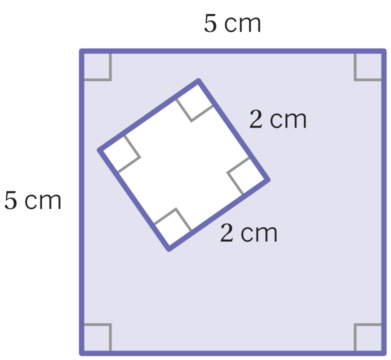
1.



2.



3.



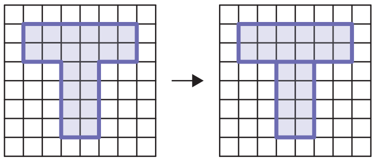
STOP

Summary

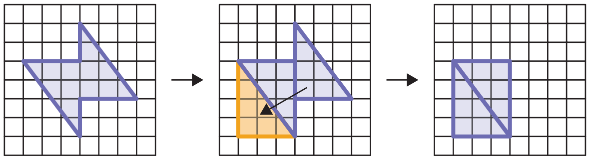
In today's lesson . . .

You saw there are several different strategies that can be used to determine the area of a shape. For instance, you can:

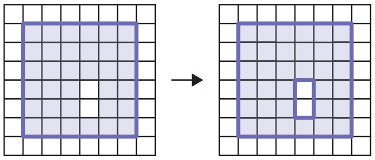
- Decompose the shape into two or more smaller shapes whose areas you know how to calculate, determine each of those areas, and then add them together.



- Decompose the shape and rearrange the pieces to form one or more other shapes whose areas you know how to calculate, determine each of those areas, and then add them together.



- Consider the shape as one with a missing piece, whose area is equal to the difference between the area of that shape and the area of the missing piece.



Area is always measured in square units. For example, when both side lengths of a rectangle are measured in centimeters, then the area is measured in square centimeters.

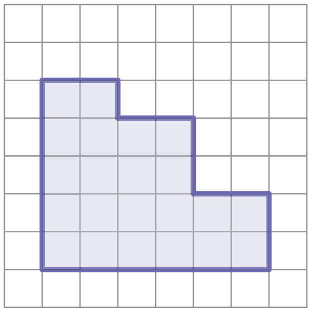
Reflect:

Name: ________________ Date: ________ Period: ________

Practice

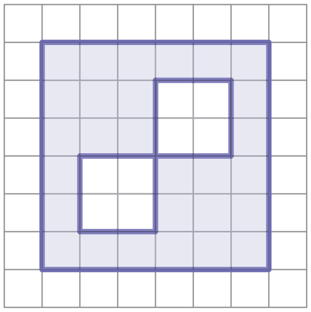
1. Each small square in these grids has an area of 1 square unit. Determine the total area of each of the shaded regions. Show or explain your thinking.

a



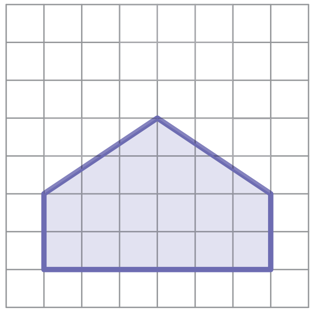
Area =

b



Area =

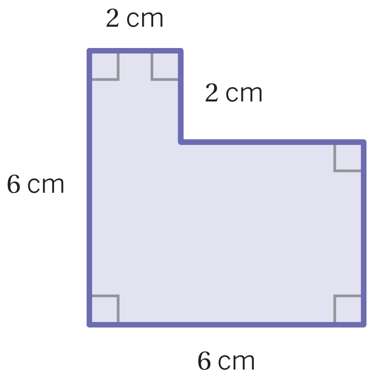
c



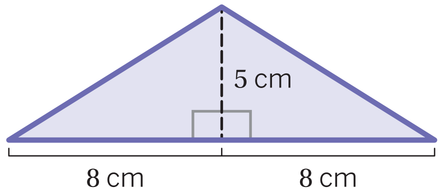
Area =

2. Determine the total area of each of the shaded regions. Show or explain your thinking.

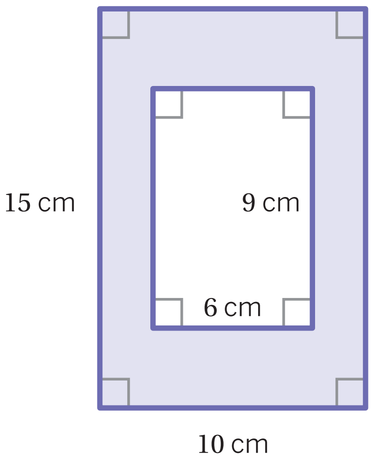
a



b

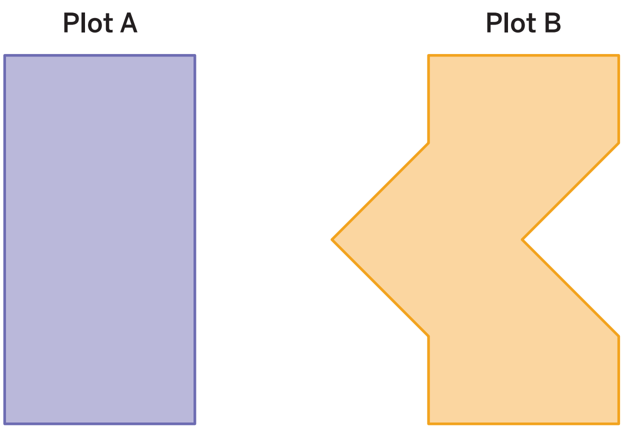


c



Name: .... Date: .... Period: ....

3. Two plots of land have very different shapes. Noah said that both plots of land have the same area. Do you agree with Noah? Explain your thinking.



4. A homeowner wants to fully cover a rectangular wall in her bathroom that measures 80 in. by 40 in. She will choose between square tiles with side lengths of either 8 in., 4 in., or 2 in. State whether you agree or disagree with each statement. Explain your thinking.

a. Regardless of the chosen tile size, she will need the same number of tiles.

b. Regardless of the chosen tile size, the area of the wall to be tiled remains the same.

c. She will need two 2-in. tiles to cover the same area as one 4-in. tile.

d. She will need four 4-in. tiles to cover the same area as one 8-in. tile.

e. If she chooses the 8-in. tiles, she will need one fourth as many tiles as she would if she chooses the 2-in. tiles.

5. Draw two quadrilaterals that have at least one pair of sides that are *parallel*. Name each quadrilateral.

Name: ________ Date: ________ Period: ________

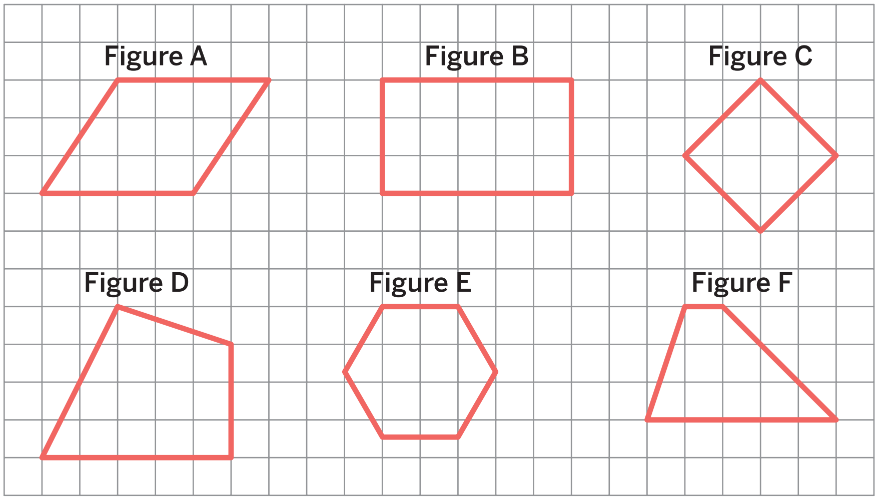
Unit 1 | Lesson 6

Parallelograms

Let's investigate the features and areas of parallelograms.

Warm-up Features of a Parallelogram

Figures A, B, and C are parallelograms. Figures D, E, and F are *not* parallelograms.



1. Study the examples and non-examples of parallelograms. What do you notice about:

   a. the number of sides the parallelograms have?

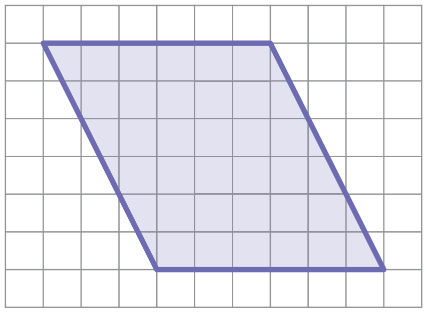
   b. the opposite sides of the parallelograms?

   c. the opposite angles of the parallelograms?

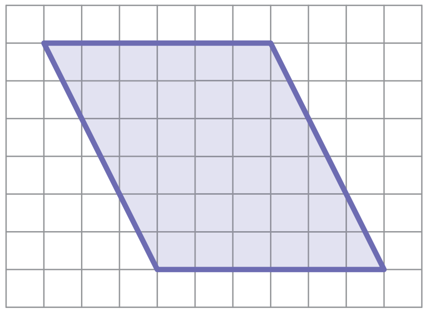
Log in to Amplify Math to complete this lesson online.

Activity 1 Decomposing and Rearranging Parallelograms

1. Refer to the parallelogram.

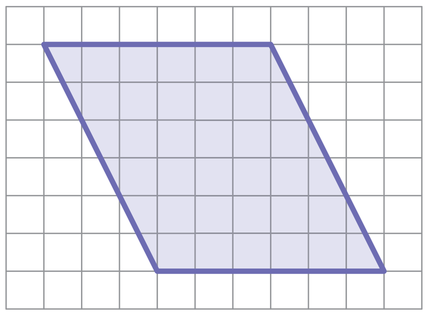


a Each small square in these grids has an area of 1 square unit. Determine the area of the parallelogram. Explain or use the grid to show the strategy you used.



b Compare answers with your group and take turns sharing your strategies.

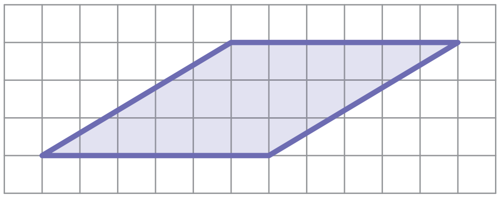
c Explain or show a different strategy than yours (used by someone else from your group) for determining the area of the parallelogram. If everyone in your group used the same strategy, work together to find a different strategy and explain or show that one.

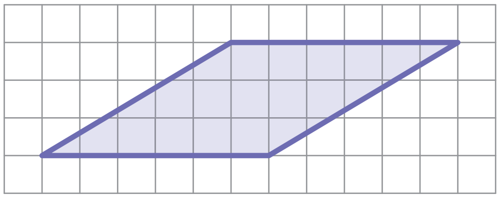


Name: ______ Date: ______ Period: ______

Activity 1 Decomposing and Rearranging Parallelograms (continued)

2. Here are two identical copies of the same parallelogram. Determine its area, and then justify your thinking by explaining or showing two different strategies.





Area:

Strategy 1:

Strategy 2:

Activity 2 Passing Parallelograms

You will be given a blank grid and a sheet containing a table. Draw any parallelogram you would like on the grid, but it *cannot* be a rectangle.

1. Determine the area of your parallelogram and record it in the table. When everyone in your group has finished, pass the drawing of your parallelogram to the person on your left.

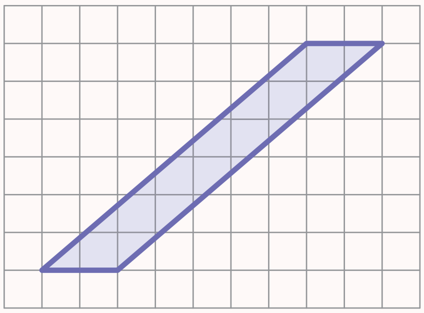
2. Determine the area of the parallelogram that was passed to you, *but do not draw on it*. Additional grids are available if you would like to redraw and mark up the parallelogram, or even cut it out. Record the area in the table alongside the name of the student who drew it.

3. Continue passing the parallelograms to the left, determining the area of each new parallelogram that is passed to you, and recording them in the table along with the name of the student who drew each one. **Note:** Each group member should see each parallelogram. Add rows to the table, as needed.

When your original parallelogram is returned to you, compare your responses and share your strategies with one another.

Are you ready for more?

Determine the area of this parallelogram.



STOP

Name: .... Date: .... Period: ....

Summary

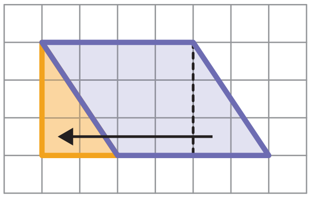
In today's lesson . . .

You revisited the defining properties of a ***parallelogram*** — a type of ***quadrilateral*** that has two pairs of parallel sides. In a parallelogram, each pair of opposite sides have the same length and each pair of opposite angles have the same measure. A *rectangle* is a special type of parallelogram in which all four angles are right angles.

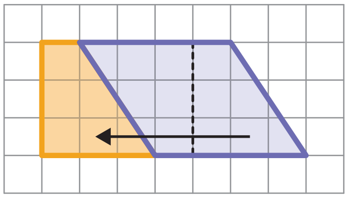
In order to determine the area of a parallelogram, you can decompose and rearrange it to calculate the area using a related rectangle:

- Decompose the parallelogram into two pieces and rearrange the pieces (using slides and flips) to form a rectangle that has the same area as the parallelogram.

  Right triangle and trapezoid

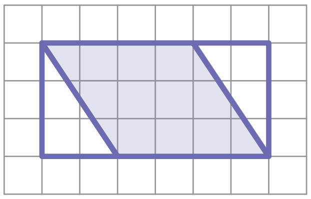
  

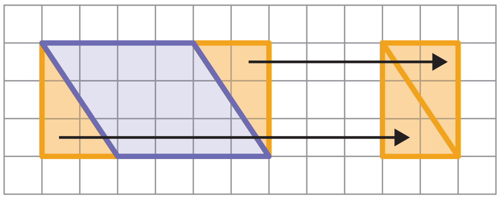
  Two right trapezoids

  The area of the related rectangle is $3 \cdot 4 = 12$ or 12 square units; therefore the area of the parallelogram is also 12 square units.
- Enclose the parallelogram in a rectangle, which is composed of two right triangles and a parallelogram. The two triangles can be composed to form a smaller rectangle, and the parallelogram's area is equal to the difference between the two rectangles' areas.

  Enclose the parallelogram in a rectangle

  The area of the parallelogram is the difference of the two rectangles.
  $(6 \cdot 3) - (2 \cdot 3) = 18 - 6 = 12$ or 12 square units.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Complete the table by deciding whether each figure is a parallelogram. For shapes that are *not* parallelograms, explain how you know they are not parallelograms.

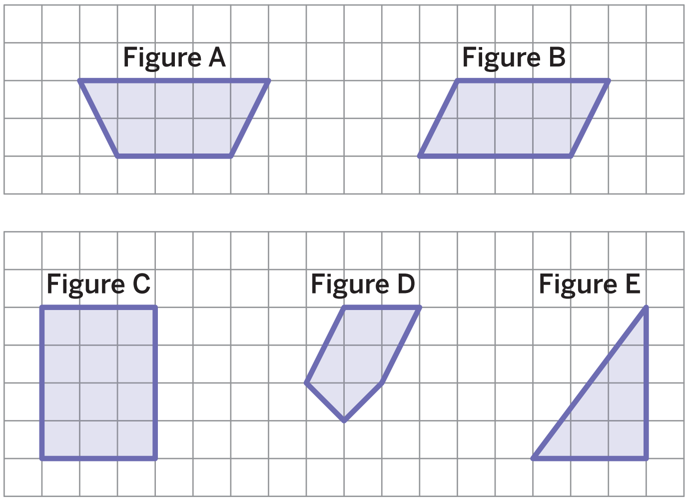
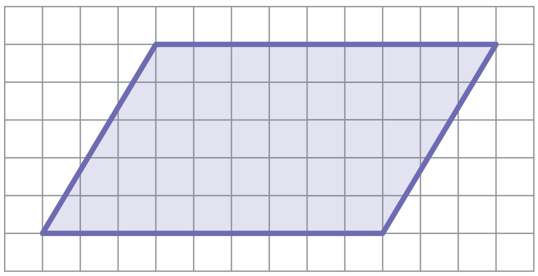


Figure	Parallelogram (Yes/No)	If not a parallelogram, how do you know?
A		
B		
C		
D		
E		

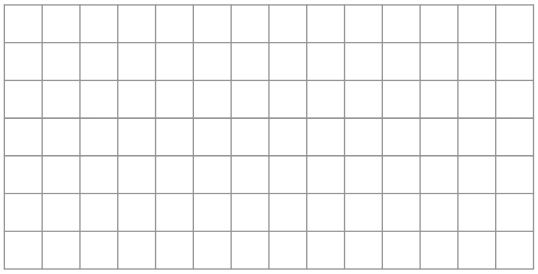
2. Refer to the parallelogram.

a Decompose and rearrange this parallelogram to form a rectangle.

b What is the area of the parallelogram? Explain or show the strategy you used.



3. Each small square in these grids has an area of 1 square unit. Draw a rectangle on the grid. Then decompose and rearrange the pieces of your rectangle to draw a parallelogram on the grid that has the same area. What is the area of each of your figures?



4. Determine which shape or shapes cover the greatest area and the least area of the plane in the pattern.

Greatest area:

Least area:

5. Use your geometry toolkit to draw each quadrilateral.

a Draw two quadrilaterals that have at least two sides that are perpendicular.

b Draw two quadrilaterals that have *no* sides that are perpendicular.

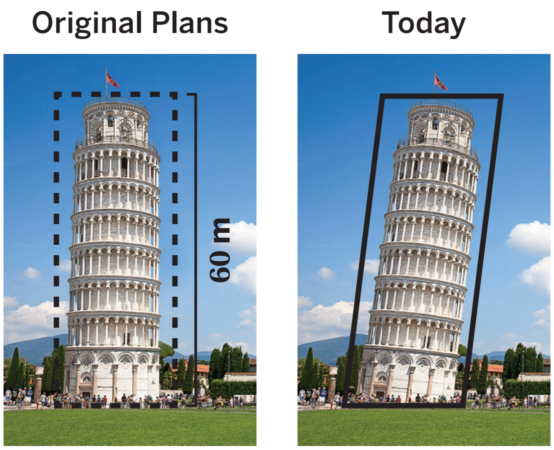
Unit 1 | Lesson 7

Bases and Heights of Parallelograms

Let's continue to investigate the areas of parallelograms.

Warm-up How Tall Is the Leaning Tower of Pisa?

The Leaning Tower of Pisa is a bell tower located in Pisa, Italy. Construction first began in 1173, but was halted multiple times due to wars, funding issues, and engineers trying to deal with the lean — which started after only three stories had been completed in 1178! Construction was finally completed in 1399, and the Leaning Tower of Pisa still stands today. It is not expected to fall for at least another 200 years, if ever.



Fedor Selivanov/Shutterstock.com

1. How would you determine how far above the ground someone would be, if they were standing on top of the tower today?

2. Define the terms *base* and *height* in your own words, and describe how each term relates to the two images of the tower.

Log in to Amplify Math to complete this lesson online.

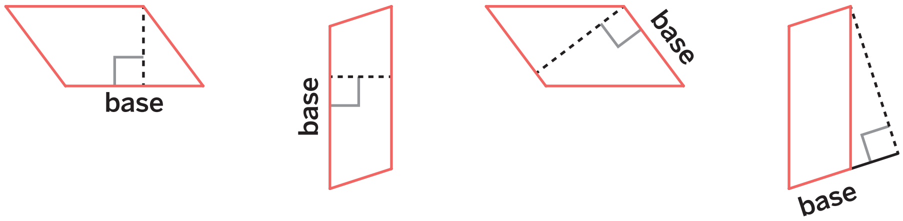
Name: ______________________ Date: __________ Period: __________

Activity 1 The Right Height

1. Think about how the base of a parallelogram relates to its height.

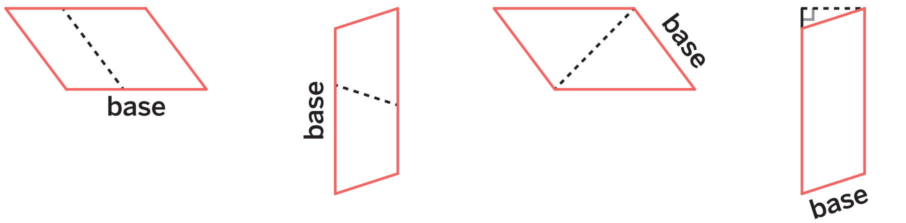
a In Figures A, B, C, and D, the dotted lines *are* a corresponding height for the labeled base.

Figure A **Figure B** **Figure C** **Figure D**



b In Figures E, F, G, and H, the dotted lines *are not* a corresponding height for the labeled base.

Figure E **Figure F** **Figure G** **Figure H**



c What must be true about a corresponding height for a given base in a parallelogram?

Discussion Support: As you share your responses, restate your classmates' reasoning to be sure you understand. Look for opportunities to challenge each other by respectfully agreeing or disagreeing.

Activity 1 The Right Height (continued)

2. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or false?	If false, make it true:
Only a horizontal side of a parallelogram can be a base.		
A base and its corresponding height must be perpendicular to each other.		
A height can only be drawn inside a parallelogram.		
A height can be drawn at any angle related to the side chosen as the base.		
For a given base, there is more than one way to draw a corresponding height.		

3. Each parallelogram is labeled to show a base b and a *potential* corresponding height h.

a Which parallelograms have a correctly labeled base and height pair?

b For each of the parallelograms that have an incorrectly labeled base and height pair, explain why the labels are not correct.

Figure J

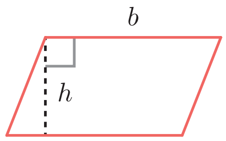


Figure K

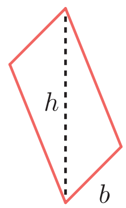


Figure L

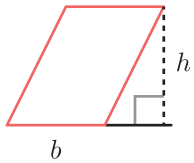


Figure M

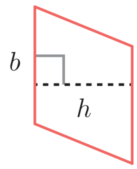
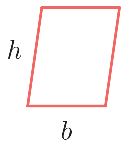


Figure N

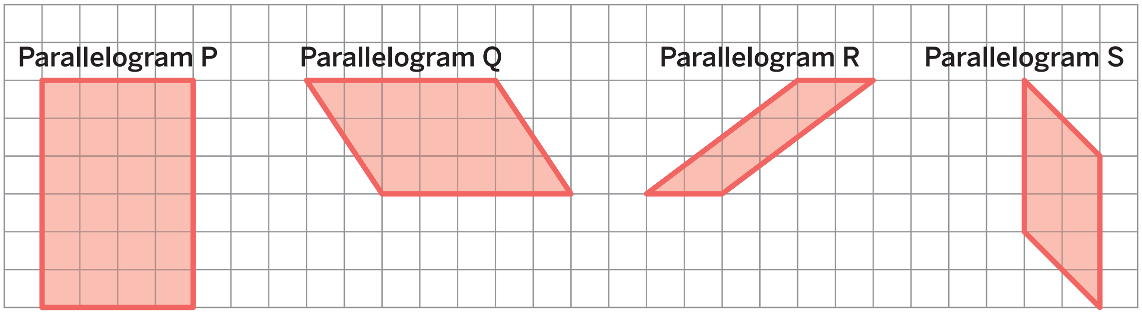


Name: .... Date: .... Period: ....

Activity 2 A Formula for the Area of a Parallelogram

1. Complete the corresponding rows of the table for each parallelogram by:

- identifying a base and a corresponding height and recording their lengths.
- determining the area of each parallelogram.



Parallelogram	Base (units)	Height (units)	Area (square units)
P			
Q			
R			
S			
Any parallelogram	b	h	

2. Complete the last row of the table by writing an expression that could be used to determine the area of *any* parallelogram, with base b and corresponding height h.

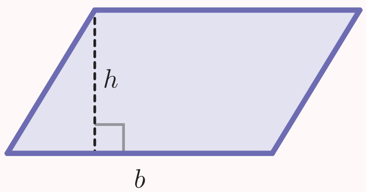
Are you ready for more?

What happens to the area of Parallelogram T if . . .

a the base is unchanged, but the height doubles? Triples? Is 100 times its original length?

b *both* the base and the height double? Triple? Each becomes 100 times their original length?

Parallelogram T



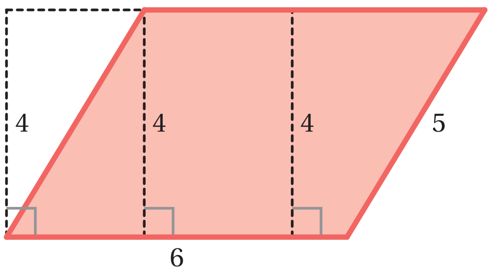
STOP

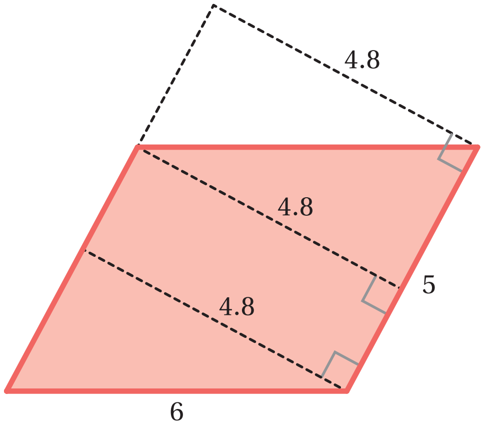
Summary

In today's lesson . . .

You saw that any of the four sides of a parallelogram can be chosen as the ***base***. Any perpendicular segment from a point on the base to the opposite side of the parallelogram represents the ***height***. There are infinitely many possible segments that can represent the height for a given base, including some that are drawn outside of the parallelogram.

These two figures show two possible bases for the same parallelogram, labeled with lengths of 6 and 5, and then three possible corresponding heights for each, labeled with lengths of 4 and 4.8.





$A = b \cdot h$

$A = 6 \cdot 4$

$A = 24$; The area is 24 square units

$A = b \cdot h$

$A = 5 \cdot 4.8$

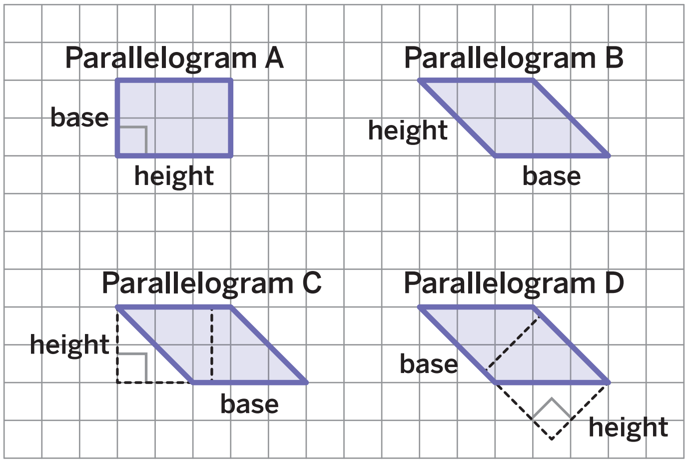
$A = 24$; The area is 24 square units

No matter which side of a parallelogram is chosen as the base, its area A is equal to the product of the length of the base b and the length of a corresponding height h.

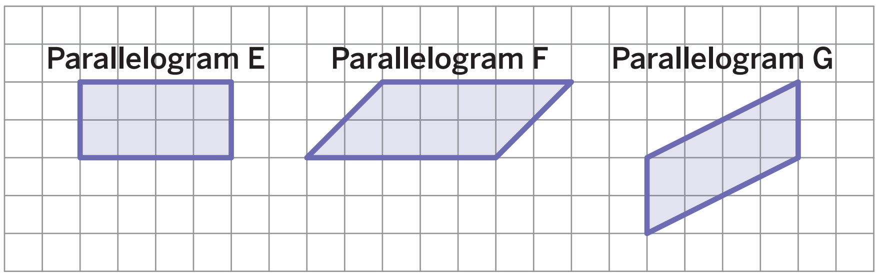
Reflect:

Name: ______ Date: ______ Period: ______

1. List the parallelograms that have a correct height labeled for the given base.



2. Each small square in these grids has an area of 1 square unit. Determine the area of each parallelogram.

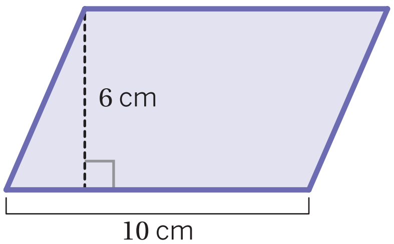


a. Area of Parallelogram E:

b. Area of Parallelogram F:

c. Area of Parallelogram G:

3. Write an expression that can be used to calculate the area of the parallelogram shown. Then use your expression to calculate the area.



Area =

Practice

Name: ______ Date: ______ Period: ______

4. Determine whether each statement is *true* or *false*. If a statement is false, write a related statement that is true.

Statement	True or False?	If false, make it true:
A parallelogram has six sides.		
Opposite sides of a parallelogram are parallel.		
A parallelogram can have one pair or two pairs of parallel sides.		
All sides of a parallelogram must have the same length.		
All angles of a parallelogram must have the same measure.		

5. A square with an area of 1 m^2 was decomposed into nine identical smaller squares. Each smaller square was then decomposed into two identical triangles.

a. What is the total area, in square meters, of six of the resulting triangles? Consider drawing a diagram to help with your thinking.

b. How many of the resulting triangles would be needed to compose a shape that has an area of $1\frac{1}{2}$ m^2?

6. Write down as many things you know or remember about a rhombus.

Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 8

Area of Parallelograms

Let's practice determining the area of parallelograms seen in the world.

Warm-up A Rhombus on the Road

Many road signs are simple geometric shapes — circles, triangles, quadrilaterals, and pentagons. This Pedestrian Crossing sign is a special type of parallelogram called a *rhombus*, which has four sides that are all the same length.

1. The sign also has four right angles. Besides being a parallelogram and a rhombus, what other shape(s) could describe the sign?

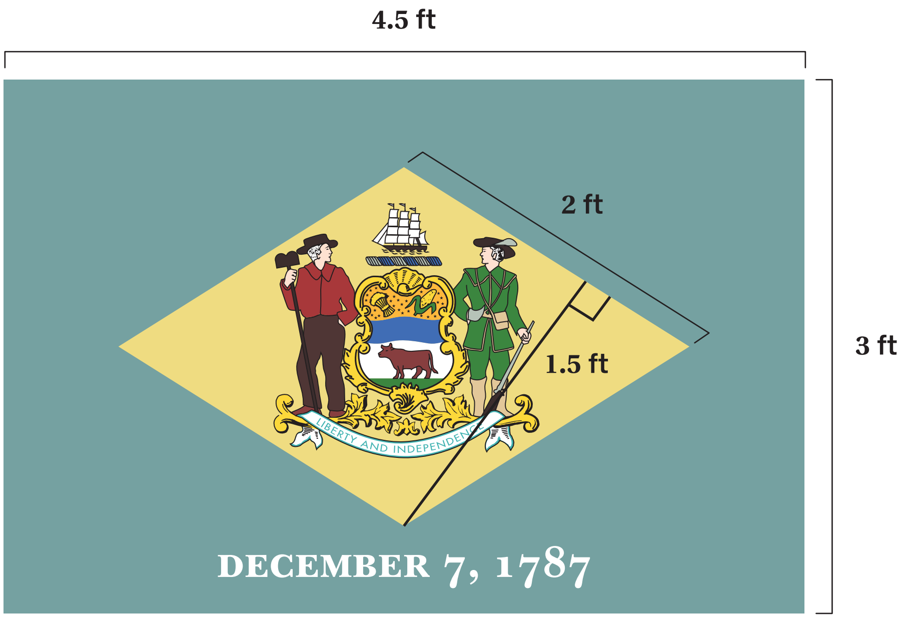
2. Each side of the sign measures 30 in. What is its area?

3. Draw another parallelogram that has the same area as the Pedestrian Crossing sign, but is neither a rhombus nor a rectangle. Be sure to label known lengths.

Log in to Amplify Math to complete this lesson online.

Activity 1 Parallelograms All Around

Delaware's state flag is shown. The official colors — colonial blue and buff yellow — represent a Revolutionary War uniform worn by General George Washington. The flag also contains the date on which Delaware became the first state to ratify the Constitution. The state's coat of arms, reading "Liberty and Independence," is displayed on top of a diamond because Delaware was once nicknamed the Diamond State. The yellow "diamond" in the center of the flag is actually a rhombus.



Public Domain

1. To create a proper rectangular flag that measures 3 ft by 4.5 ft, the rhombus would have side lengths of 2 ft and a perpendicular distance across of 1.5 ft. Determine how much of each color fabric is used to make the two main parts of the flag. Explain or show your thinking.

   a. Yellow rhombus

   b. Blue rectangle

Name: .... Date: .... Period: ....

Activity 1 Parallelograms All Around (continued)

2. A local charity organization has placed drop boxes for donations around town, such as the one shown here.

a The base of the logo on a drop box measures approximately 25 cm, and the height measures approximately 15 cm. About how many square centimeters of space does the logo take up on the side of a drop box?

b A prototype for printing the logo on the side of a transport and delivery truck takes up about 735 in^2 of space, and measures about 35 in. horizontally across the bottom edge. What is the corresponding height of the logo for the truck?

3. Handicapped parking spaces are given extra clearance from the curb, and a "no parking" area is often marked in between to allow a wheelchair to enter and exit a vehicle safely. The slanted lines marking the "no parking" space shown here form 9 parallelograms and 2 right triangles (each of which is exactly half of one parallelogram).

Amy Sroka/Amplify

If the length of the parking space is 18 ft (the minimum required), and the "no parking" area covers 90 ft^2, how far is the right side of this handicapped parking space from the curb?

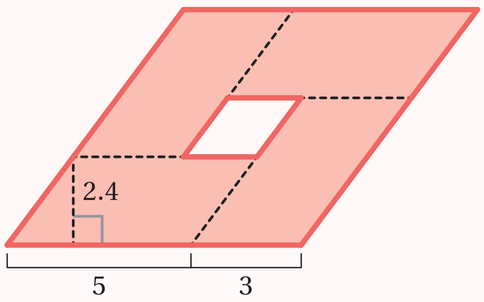
Activity 2 Drawing Parallelograms

Use the grid to draw two different parallelograms, labeled as P and Q, that meet the following criteria:

- **They both have the same area of 20 square units.**
- **Neither of the parallelograms is a rectangle.**
- **The two parallelograms do not have any side lengths in common with one other.**

Are you ready for more?

The shape shown is composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches. What is the area of the unshaded parallelogram in the middle? Explain or show your thinking.



STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw examples of several different parallelograms in the real world. Given their dimensions, you calculated their areas, bases, and/or heights.

The formula for the area of a parallelogram can be used to determine *any* of the three measures involved, not just the area. For example:

- If you know the area and the length of the base, you can determine the length of a corresponding height.

Base	Height	Area
5 cm	?	60 cm^2

$$A = b \cdot h$$
$$60 = 5 \cdot ?$$
$$60 \div 5 = 12$$
The height is 12 cm.

- Similarly, if you know the area and the measure of a height, you can determine the length of the base.

Base	Height	Area
?	8 in	16 in^2

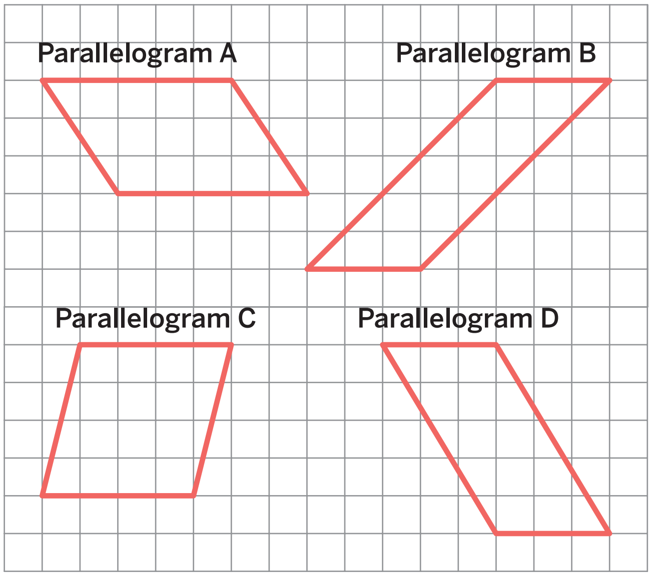
$$A = b \cdot h$$
$$16 = ? \cdot 8$$
$$16 \div 8 = 2$$
The base is 2 in.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. Three of these parallelograms have the same area. Which one has a *different* area than the others?



2. The base lengths b and corresponding heights h of four different parallelograms are listed. Which base-height pair represents the parallelogram with the greatest area?

A. $b = 4, h = 3.5$

B. $b = 0.8, h = 20$

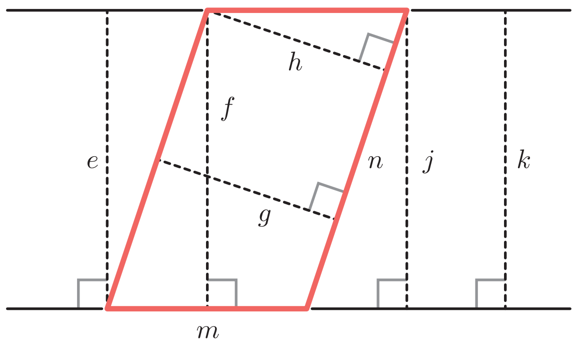
C. $b = 6, h = 2.25$

D. $b = 10, h = 1.4$

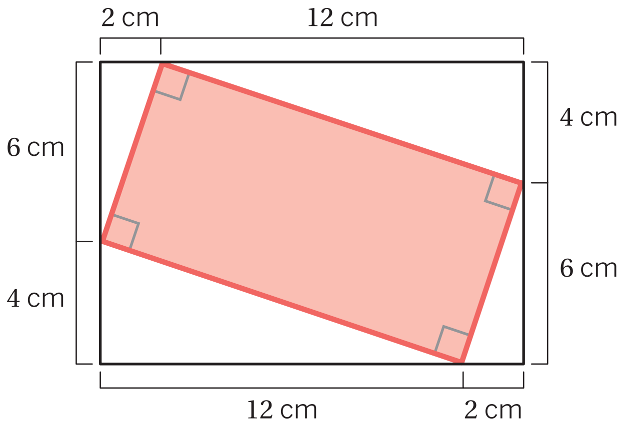
3. Two opposite faces of the Dockland Building in Hamburg, Germany are shaped like identical parallelograms. The length of the face of the building shown is approximately 86 m along the bottom, and its height is approximately 55 m from the bottom to the top. Using this information, estimate the area of this face of the building.

foto-select/Shutterstock.com

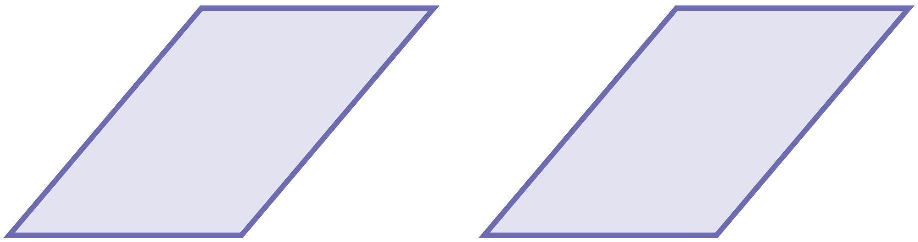
4. The parallelogram shown has side lengths m and n. List *all* of the lengths that represent a corresponding height for the base m.



5. Determine the area of the shaded region. Show or explain your thinking.

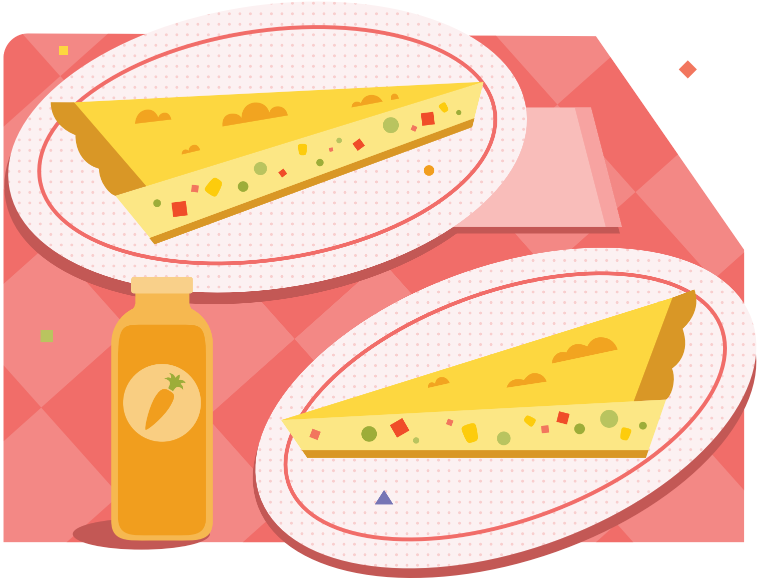


6. Draw a straight line on *each* copy of this quadrilateral, where each line partitions the shape into equal-sized halves. Show a different partition for each shape.



Unit 1 | Lesson 9

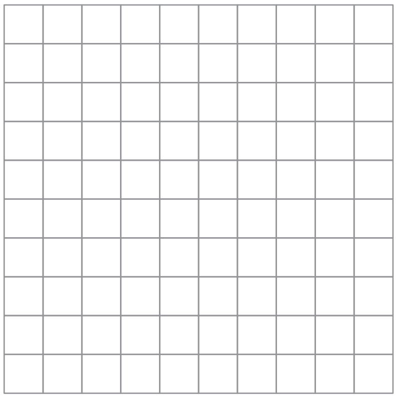
From Parallelograms to Triangles

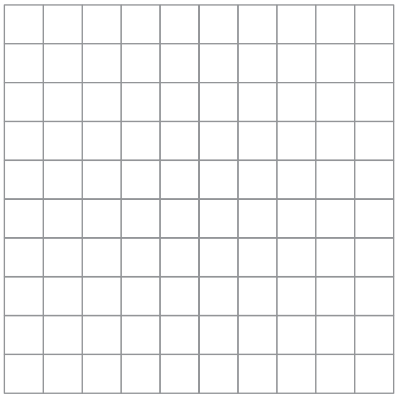


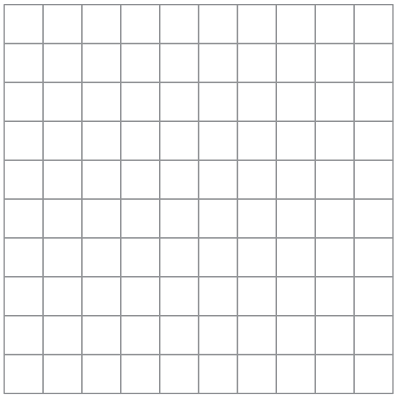
Let's use what we know about parallelograms to determine the area of triangles on grids.

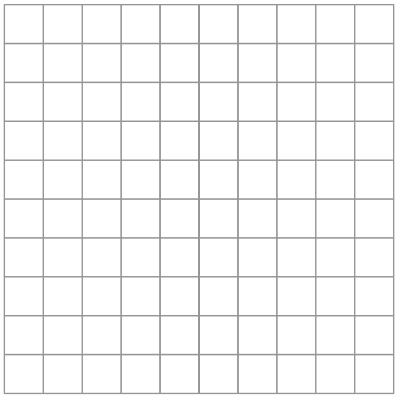
Warm-up Composing Parallelograms

You will be given two identical triangles on a grid. Create as many *different* parallelograms as you can using both triangles. Trace the perimeter of each different parallelogram on one of the grids. You might not use all the grids.









Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Decomposing Parallelograms

Refer back to the Warm-up and label three of the parallelograms as A, B, and C.

1. Assume each grid square has an area of 1 square unit. Calculate the area of each parallelogram.

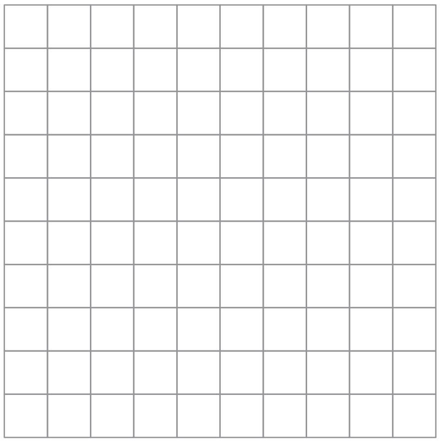
a Parallelogram A:

b Parallelogram B:

c Parallelogram C:

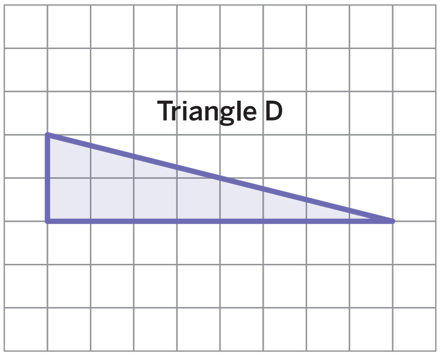
2. What is the area of one of the original triangles? Show or explain your thinking.

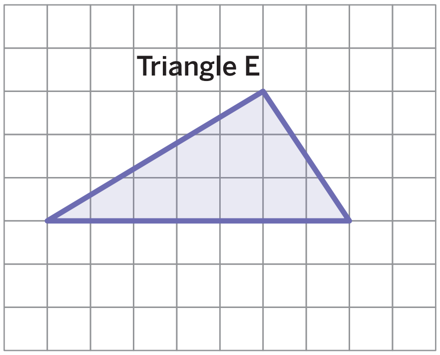
3. Draw a different triangle that has the same area as one of the triangles from the Warm-up. Show or explain how you know it has the same area.



Activity 2 Determining the Area of Triangles

On this page and the next page, four different triangles, labeled D, E, F, and G, are shown on grids. Each small square in these grids has an area of 1 square unit. Choose *three* of the triangles and determine their areas. Use at least two different strategies altogether. Show or explain your thinking for each triangle you chose.

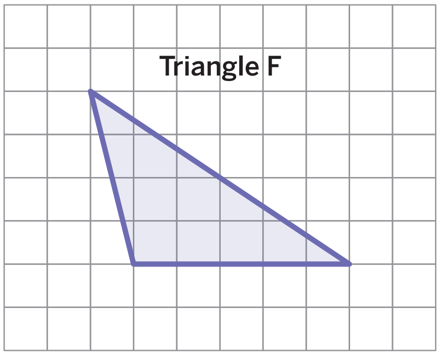
1. 

2. 

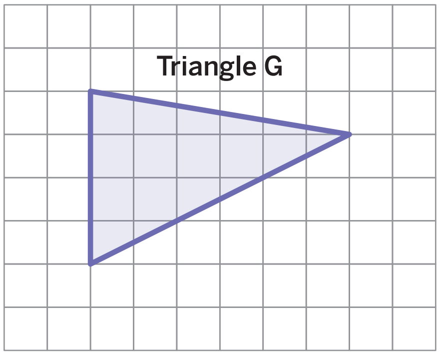
Name: ______ Date: ______ Period: ______

Activity 2 Determining the Area of Triangles (continued)

3.



4.



Reflect: How did patterns provide a structure for finding the area of parallelograms and triangles?

STOP

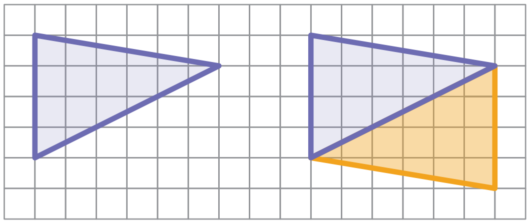
Summary

In today's lesson . . .

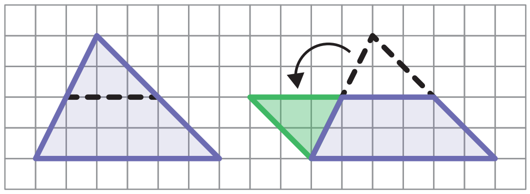
You saw several ways to reason about the area of a triangle using what you know about composition and decomposition of shapes and the area of parallelograms.

Here are three possible strategies to determine the area of a triangle on a grid:

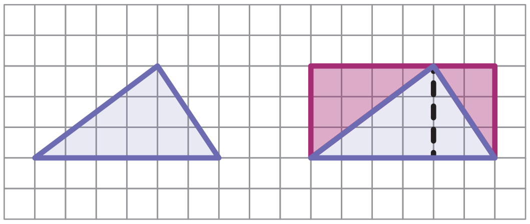
- Make a copy of the triangle and compose the two identical triangles to form a parallelogram — for a right triangle, this will be a rectangle. Because the two triangles have the same area, each triangle has an area that is exactly half the area of the parallelogram.



- Decompose the triangle and rearrange the pieces to form a parallelogram. Because the triangle and the parallelogram are made up of exactly the same pieces, their areas are equal.



- Enclose the triangle in a large rectangle that can be decomposed into two smaller rectangles. This also decomposes the triangle into two smaller triangles. Each of these smaller triangles has half the area of its enclosed rectangle. The sum of the two smaller triangles' areas is equal to the area of the original triangle.



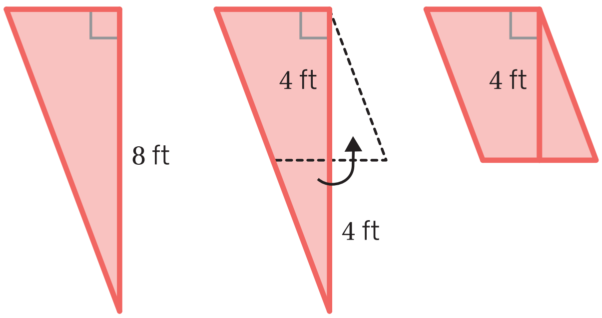
Reflect:

Name: ________________ Date: ________ Period: ________

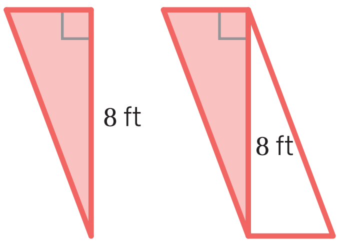
Practice

1. To determine the area of a given right triangle, Diego and Jada used different strategies.

a. Diego drew a line through the midpoints of the two longer sides, which decomposed the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes to form a parallelogram. Explain how Diego could use his parallelogram to determine the area of the triangle.

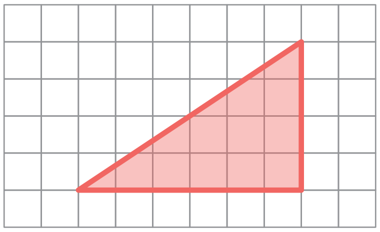


b. Jada made an identical copy of the triangle and used the two identical copies to compose a different parallelogram. Explain how Jada could use her parallelogram to determine the area of the triangle.

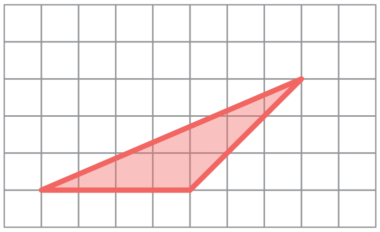


2. Each small square in these grids has an area of 1 square unit. Determine the area of each triangle.

a.



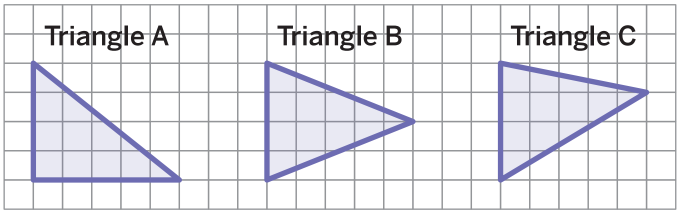
b.



Practice

Name: ______ Date: ______ Period: ______

3. Which of these triangles has the greatest area? Show or explain your thinking.



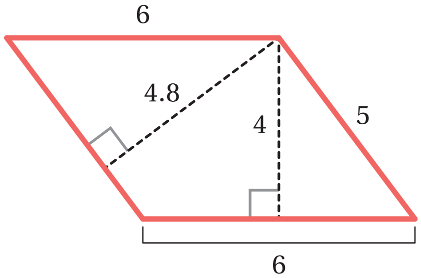
4. Solve the following problems involving the base, height, and area of parallelograms.

   a. A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

   b. A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

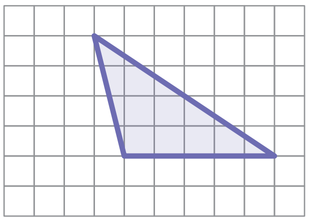
   c. A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the length of that base?

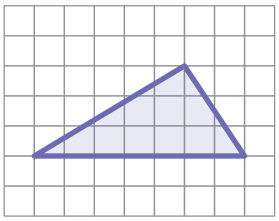
5. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



   A. 6 units
   B. 4.8 units
   C. 4 units
   D. 5 units

6. Using what you know about parallelograms, label what you think would be the base and height for each of these triangles.

   a. 

   b. 

Name: ______________________ Date: ____________ Period: ____________

Unit 1 | Lesson 10

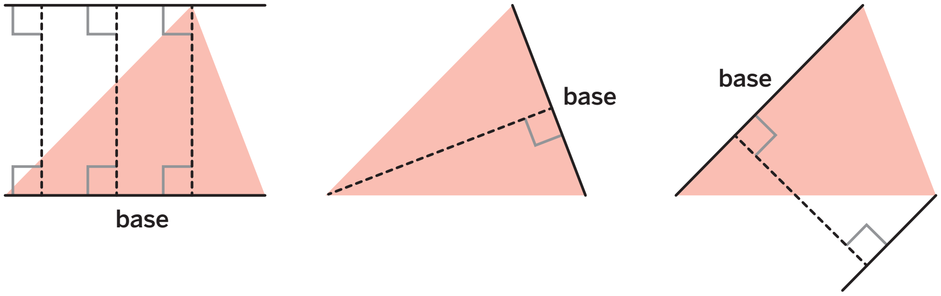
Bases and Heights of Triangles

Let's find the bases and heights of triangles.

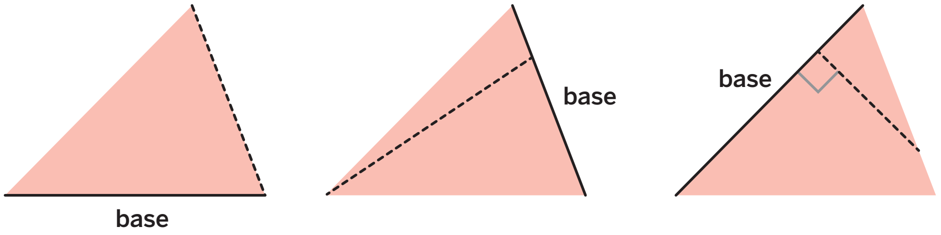
Warm-up Base and Height Pairs

Study the examples and non-examples of bases and heights in a triangle.

These dashed segments represent heights of the triangles.



These dashed segments do *not* represent heights of the triangles.



Based on these examples and nonexamples, how would you define the *base* and *height* of a triangle?

Log in to Amplify Math to complete this lesson online.

Activity 1 The Truth About Bases and Heights

Refer to the examples and non-examples of bases and heights for triangles from the Warm-up.

1. Determine whether each statement is *true* or *false*. Place a check mark in the appropriate column.

Statement	True	False
Any side of a triangle can be the base.		
A height must always be one of the sides of a triangle.		
A height that corresponds to the base of a triangle can be drawn at any angle to the base.		
For a chosen base, there is only one possible height that can be drawn.		
A height must have an endpoint at a vertex of the triangle or along the line parallel to the base that extends from the opposite vertex.		

2. Choose one of the statements you identified as false, and explain why it is false.

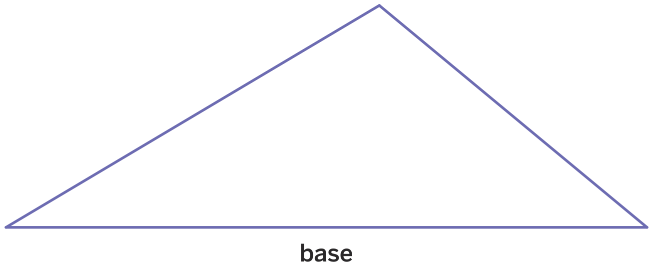
3. Using your chosen statement from Problem 2, alter the statement so that it is true. Rewrite the true statement here.

Name: ____________ Date: ____________ Period: ____________

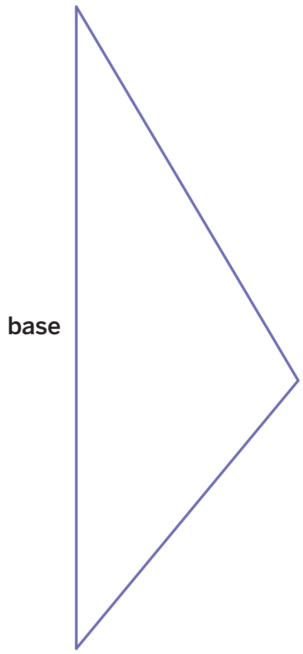
Activity 2 Hunting for Heights

1. Refer to Triangles A, B, C, and D. Draw a height that corresponds to each given base. Consider using an index card to help you.

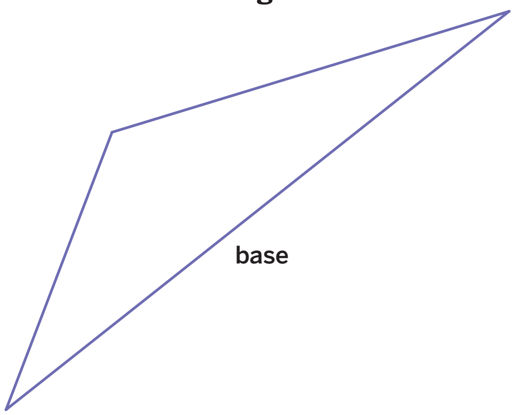
Triangle A



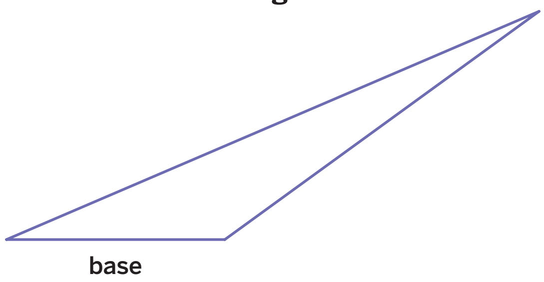
Triangle B



Triangle C



Triangle D

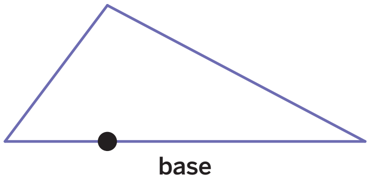


Activity 2 Hunting for Heights (continued)

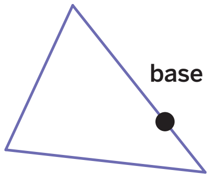
2. Refer to Triangles E–K. Continue to draw the heights for the given bases.

a For Triangles E–G, draw a height that intersects the base at the given point.

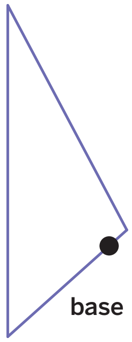
Triangle E



Triangle F

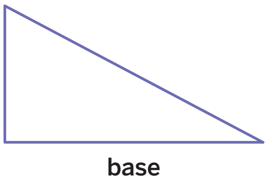


Triangle G

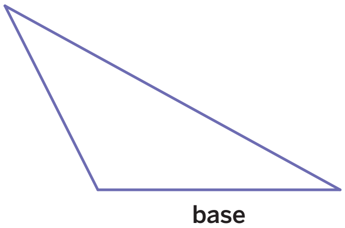


b For Triangles H–K, any corresponding height can be drawn.

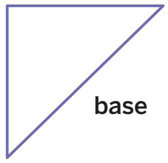
Triangle H



Triangle J



Triangle K



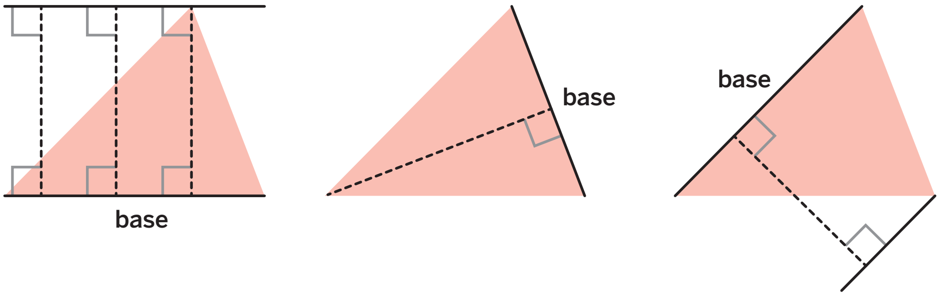
STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw that any side of a triangle can be chosen as the ***base*** of the triangle. Once the base has been identified, there are many possible segments that can be drawn for the ***height*** of the triangle. Most commonly this is the segment drawn perpendicular to the base from the *opposite vertex*. A height can also extend outside of the triangle – even entirely outside – from any point along the line containing the base to a line parallel to the base that contains the opposite vertex, intersecting both lines at right angles.



Even though any side of a triangle can be a base, some base and height pairs can be more easily determined than others, so it helps to choose strategically. For example:

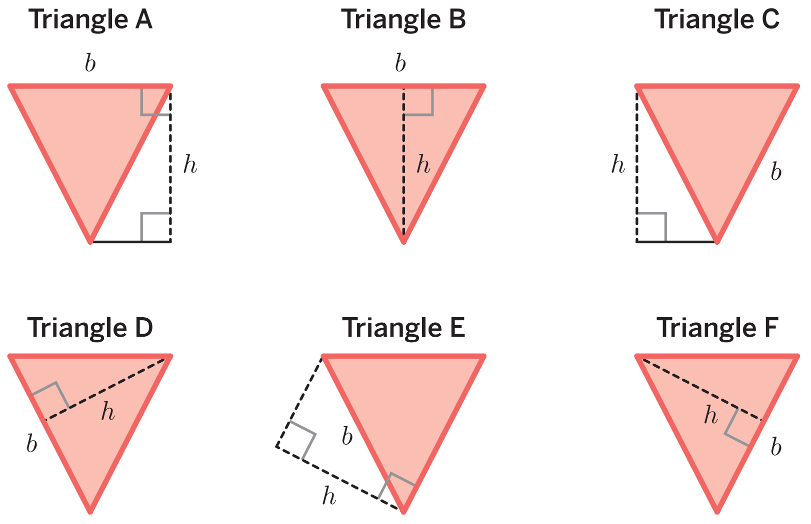
- When working with a right triangle, it often makes sense to use the two sides that form the right angle as the base and the height.
- When working on a grid, selecting a side that aligns to a horizontal or vertical grid line ensures that a perpendicular height can also be drawn along a grid line.

Reflect:

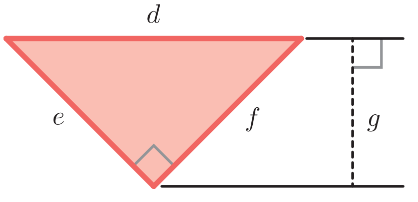
Practice

Name: ______ Date: ______ Period: ______

1. Name *all* the triangles for which a corresponding height h for the given base b is correctly identified.



2. Refer to the right triangle shown. Name a corresponding height for each indicated base.

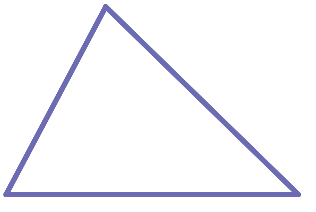


a) Base d

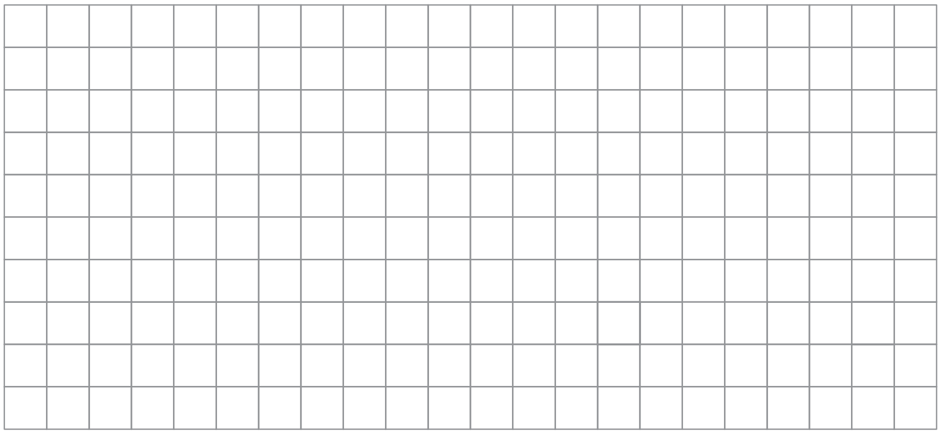
b) Base e

c) Base f

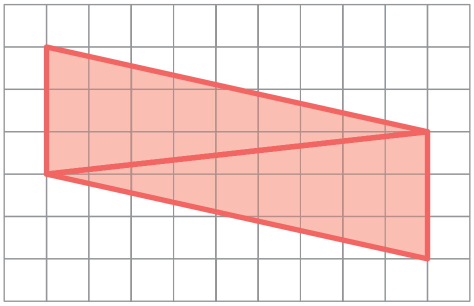
3. Identify a base of the triangle and draw a corresponding height for your chosen base. Explain how to identify the base and how to draw the height.



4. Draw two different parallelograms that have the same area. Label a corresponding base and height for each parallelogram and explain how you know the areas are the same.



5. Andre drew a diagonal segment connecting two opposite vertices of this parallelogram. Select *all* the true statements about the two triangles that are formed.



A. Each triangle has two sides that are 3 units long.

B. Each triangle has a side that is the same length as the diagonal segment.

C. Each triangle has one side that is 3 units long.

D. The triangles are not identical copies of one another.

E. The triangles have the same area.

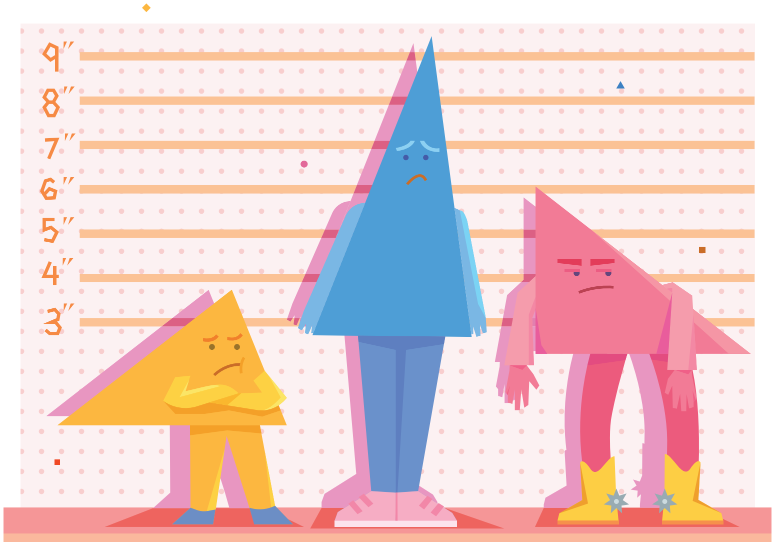
F. Each triangle has an area that is half the area of the parallelogram.

6. How can drawing a triangle on a grid help you identify a valid base and height pair in the triangle?

Unit 1 | Lesson 11

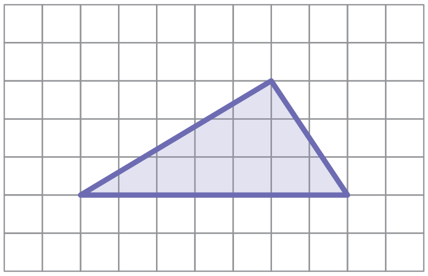
Formula for the Area of a Triangle

Let's write and use a formula to determine the area of any triangle.



Warm-up Same Bases, Same Heights

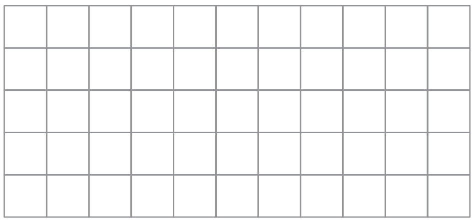
1. Label the base and draw a corresponding height for the triangle. Then write the measurement for each.



Base: .... units

Height: .... units

2. Draw a parallelogram with the same base and height measurements that you identified for the triangle in Problem 1.

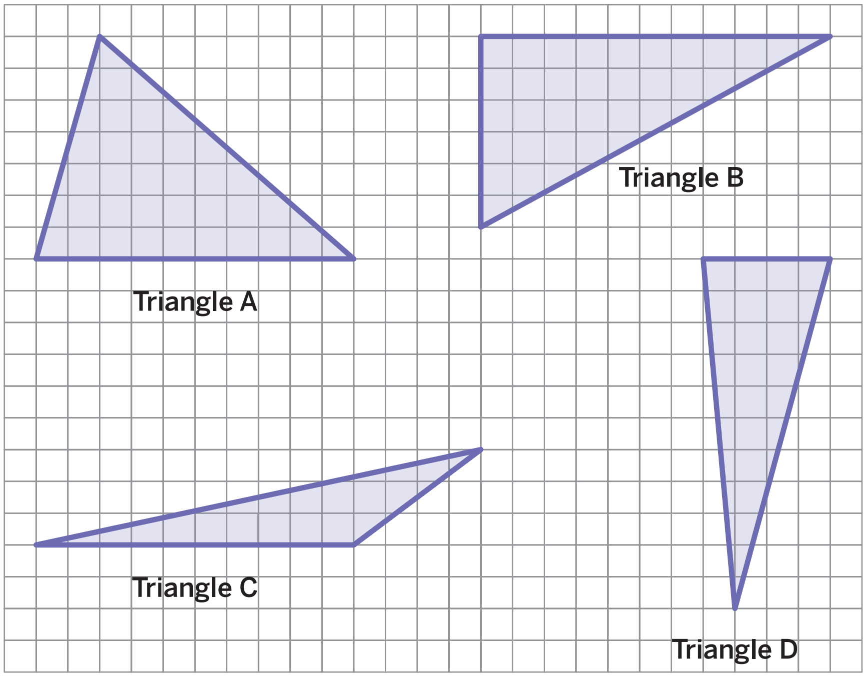


Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 The Formula for the Area of a Triangle

1. Label a base and corresponding height pair for each of the four triangles shown.



2. Complete the table by recording the following measurements of Triangles A, B, C, and D:

- The length of the base you identified
- The corresponding height
- The area

Triangle	Base (units)	Height (units)	Area (square units)
A			
B			
C			
D			
Any triangle	b	h	

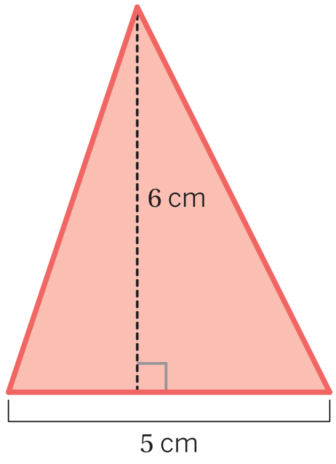
3. Complete the last row of the table by writing an expression for the area of any triangle, using b for the length of the base and h for the corresponding height.

Activity 2 Applying the Formula for the Area of a Triangle

Write the base and height measurements that can be used to calculate the area of each triangle. Then calculate the area and show or explain your thinking.

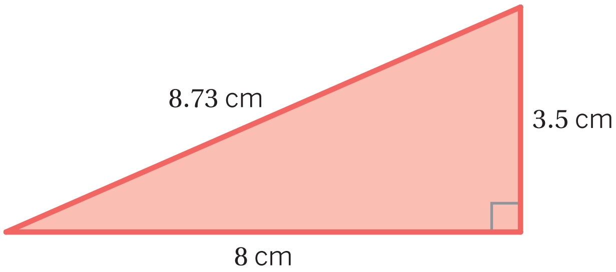
1. Base:
   Height:
   Area:
   My thinking:

Triangle E



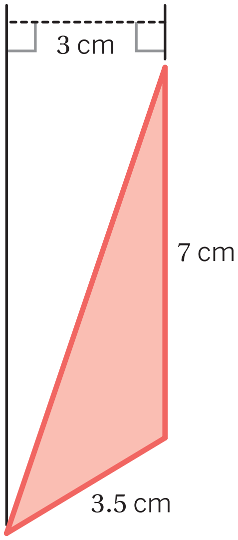
2. Base:
   Height:
   Area:
   My thinking:

Triangle F



3. Base:
   Height:
   Area:
   My thinking:

Triangle G



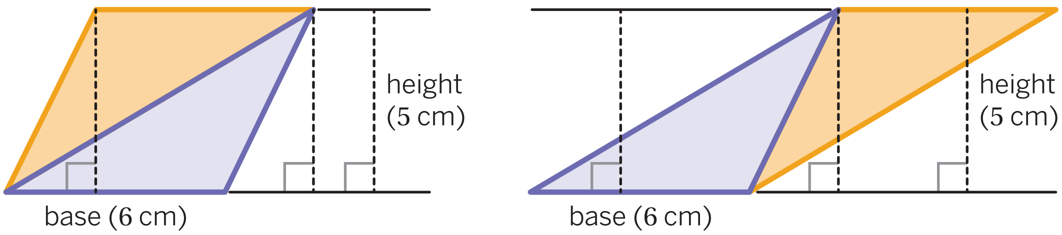
STOP

Name: ______________ Date: ______________ Period: ______________

Summary

In today's lesson . . .

You saw that the base and height pairs of a triangle are closely related to those of a parallelogram. Recall that two identical copies of a triangle can be composed to form a parallelogram. Regardless of how they are composed, the original triangle and the resulting parallelogram will have a common side length. Labeling these sides as the bases, the corresponding heights will be the same, no matter where those segments are drawn.



Using this relationship between triangles and parallelograms, the area of such a triangle will always be equal to exactly half the area of the corresponding parallelogram.

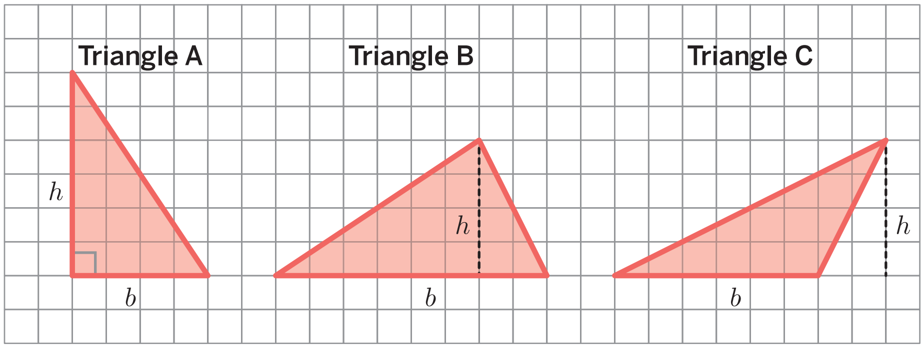
Area of Parallelogram	Area of Triangle
$A = b \bullet h$ $A = (6) \bullet (5)$ $A = 30$ The parallelogram has an area of 30 cm^2.	Half the area of the parallelogram, so; $A = \frac{1}{2}\ b \bullet h$ $A = \frac{1}{2} \bullet (6) \bullet (5)$ $A = 15$ The triangle has an area of 15 cm^2.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. For each triangle, a base b and its corresponding height h are labeled.



a. Determine the area of each triangle.

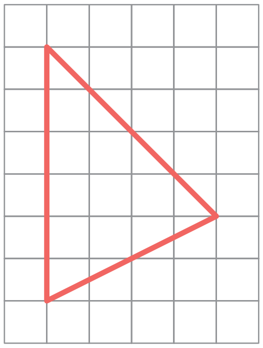
Triangle A:

Triangle B:

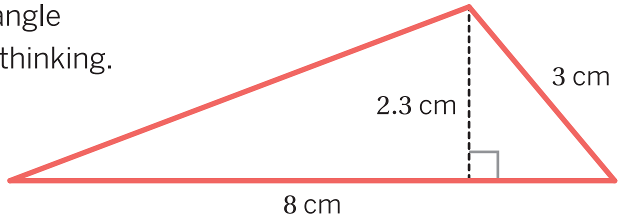
Triangle C:

b. How is the area of *any* triangle related to the length of a chosen base and its corresponding height?

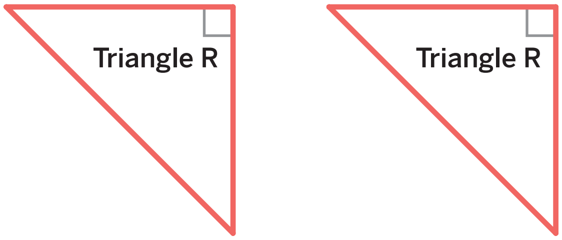
2. Determine the area of the triangle. Show or explain your thinking. To help with your thinking, carefully consider which side of the triangle to use as the base.



3. Determine the area of the triangle shown. Show or explain your thinking.



4. Triangle R is a right triangle. Is it possible to compose a parallelogram that is *not* a square using two identical copies of Triangle R? If so, show or explain how. If not, explain why not.



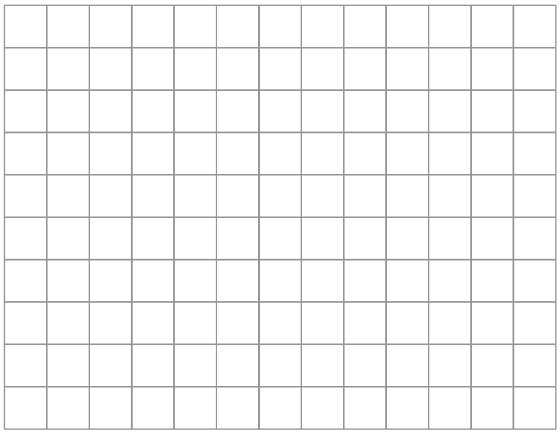
5. Determine the stated measurement for each parallelogram described.

a A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

b A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

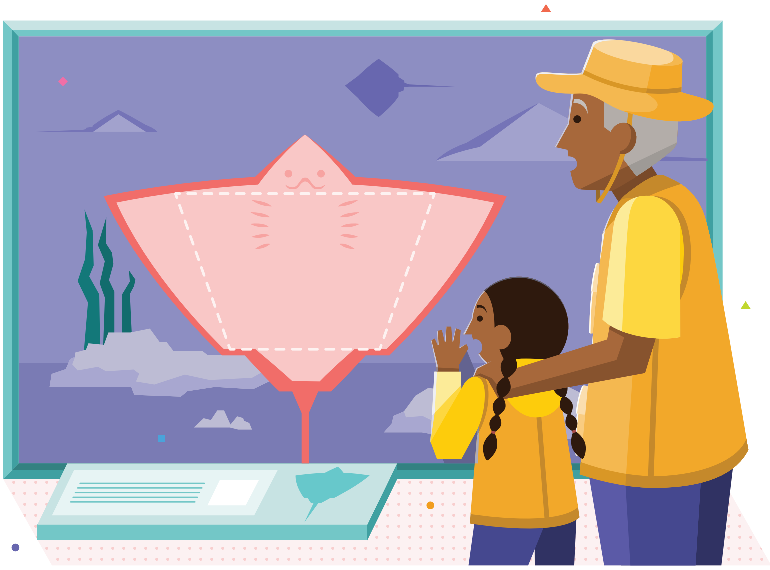
c A parallelogram has an area of 7 square units and the height corresponding to the chosen base is $\frac{1}{4}$ units. What is the length of the base?

6. On the grid, show how a trapezoid can be composed using two or more parallelograms and/or triangles.



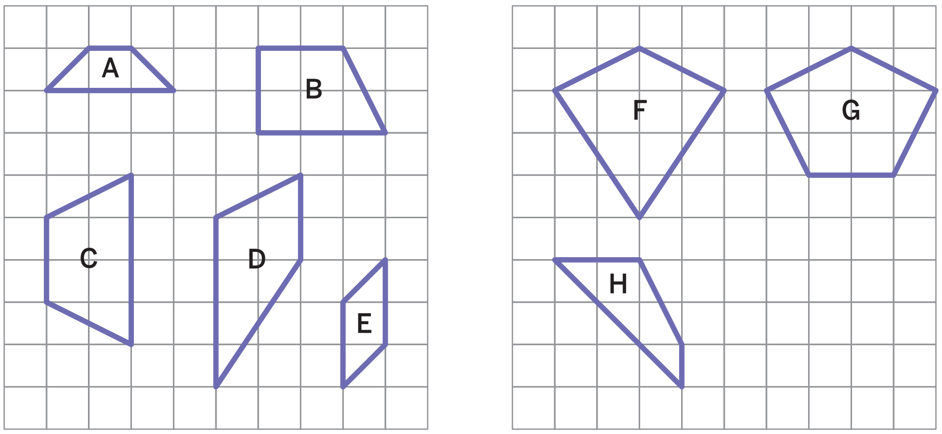
From Triangles to Trapezoids

Let's apply what we know about triangles and parallelograms to trapezoids.



Warm-up Features of a Trapezoid

Study the figures on the grid. Figures A–E are trapezoids. Figures F–H are *not* trapezoids.



1. What do you notice about:
   a. the number of sides the trapezoids have?
   b. the opposite sides of the trapezoids?

2. Choose one shape from the non-examples and explain why it is *not* a trapezoid.

3. What do you notice about Figure E, compared to Figures A–D?

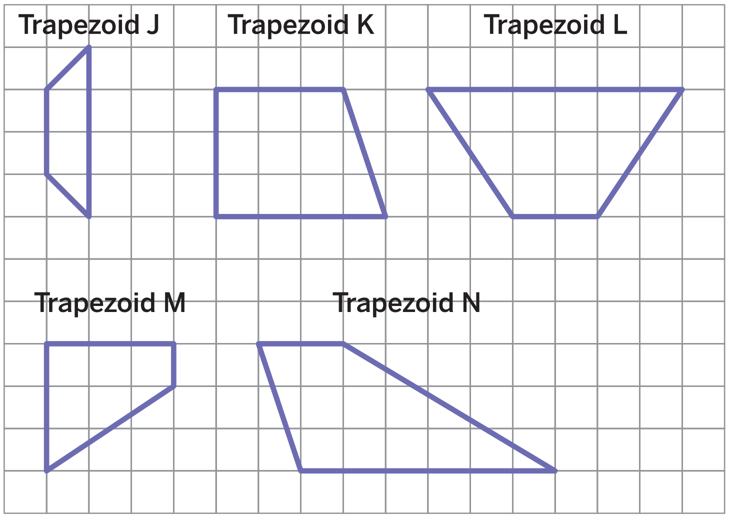
Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Decomposing Trapezoids

Even though they have just four sides, quadrilaterals are still studied by many mathematicians, including Santana Afton, who explores connections between quadrilaterals and prime numbers.

For now, refer to Trapezoids J–N. Show how each of the following decompositions can be created for at least one of the trapezoids by drawing partitioning lines. For each decomposition, write the letters of the corresponding trapezoids, based on your drawings.



1. Two triangles:
2. A parallelogram and a triangle:
3. A parallelogram and two identical triangles:
4. A parallelogram and two different triangles:

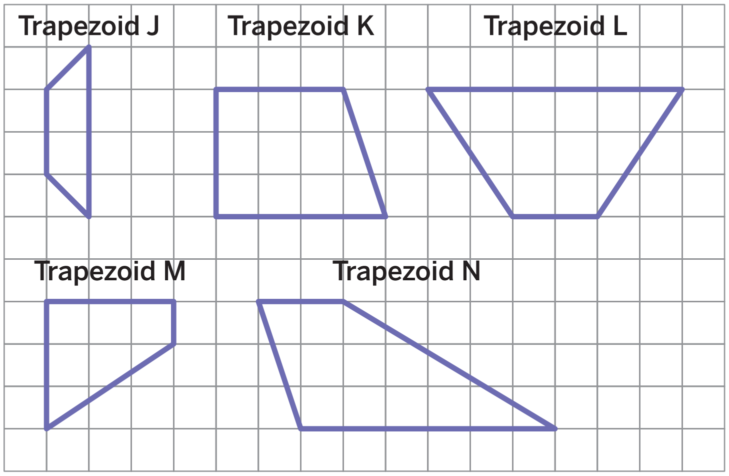
Featured Mathematician

Santana Afton

Santana Afton studies geometry and topology at Georgia Tech. As part of their research, Afton explores what are called "generalized quadrangles." These are structures that contain only quadrilaterals (that is, no triangles), and which can be described with two whole numbers. Beyond their research, Afton enjoys mentoring undergraduate and high school students interested in mathematics.

Activity 2 Area of Trapezoids

The same trapezoids from Activity 1 are shown. Determine the area of each trapezoid. Then complete the table by recording the base and height measurements for each.

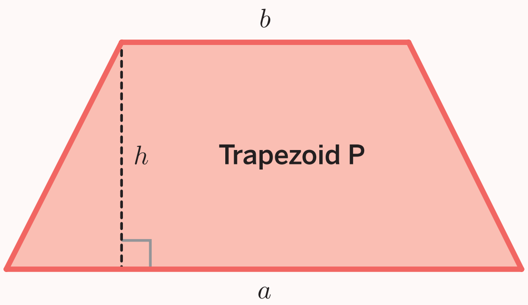


Trapezoid	Base 1 (units)	Base 2 (units)	Height (units)	Area (square units)
J				
K				
L				
M				
N				

Are you ready for more?

Trapezoid P shown here has bases with unknown lengths of a units and b units, and an unknown height of h units.

Add a row to your table, completing each of the first three columns with an appropriate letter and the last column with an expression. This expression will be the formula for the area of *any* trapezoid.



Hint: Consider looking for relationships or patterns among the values already in your table from Trapezoids J–N, or decompose Trapezoid P into shapes whose areas you can determine.

STOP

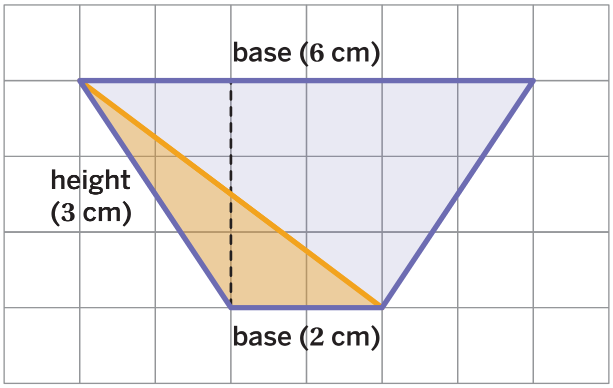
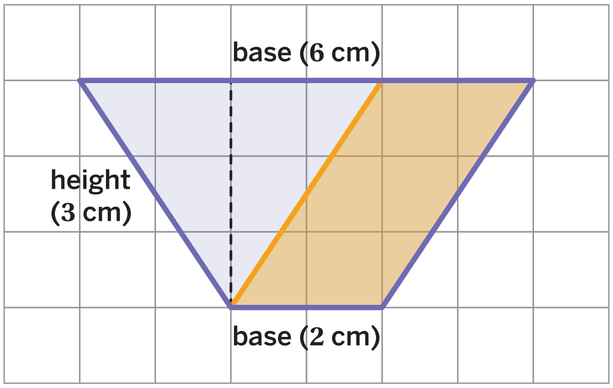
Name: ____________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You reviewed the defining characteristics of a ***trapezoid*** — a quadrilateral that has at least one pair of parallel sides, which are called its bases.

You expanded on your understanding of the area of rectangles, parallelograms, and triangles, in order to decompose trapezoids and determine their areas. There are often multiple ways to decompose the same trapezoid.

Decomposition	Area
	Area is the sum of the area of two triangles. $A = \frac{1}{2}(2 \cdot 3) + \frac{1}{2}(6 \cdot 3)$ $A = \frac{1}{2}(6) + \frac{1}{2}(18)$ $A = 3 + 9$ $A = 12$ The area of the trapezoid is 12 cm^2.
	Area is the sum of the areas of the parallelogram and triangle. $A = (2 \cdot 3) + \frac{1}{2}(4 \cdot 3)$ $A = 6 + \frac{1}{2}(12)$ $A = 6 + 6$ $A = 12$ The area of the trapezoid is 12 cm^2.

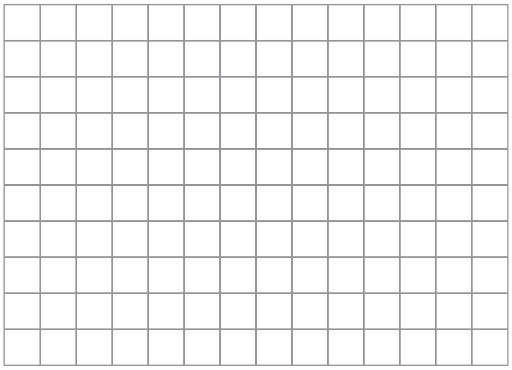
Reflect:

Name: ______________________ Date: ____________ Period: __________

1. Draw a trapezoid and the corresponding partition line(s) on each grid based on these descriptions. Then determine the area of each trapezoid.

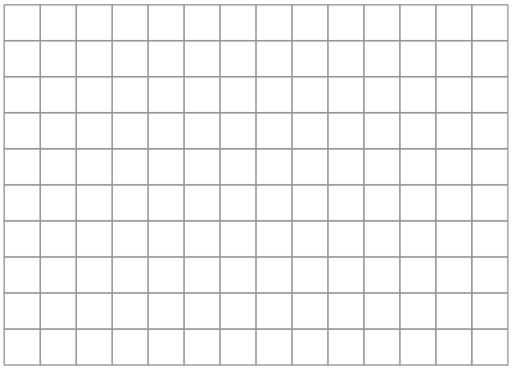
   a. A trapezoid that can be decomposed into exactly one parallelogram and one triangle with a single partition line.

      Area:

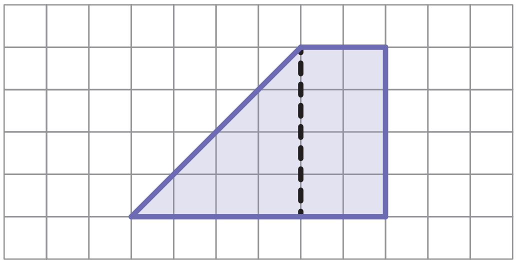


   b. A trapezoid that can be decomposed into exactly one parallelogram and two identical triangles using two partition lines.

      Area:



2. To help determine the area of this trapezoid, Clare decomposed it into two shapes by drawing the dashed line shown.

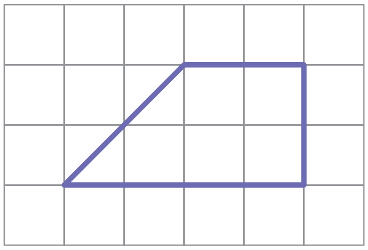


   a. Clare thinks that the two resulting shapes have the same area. Do you agree? Explain your thinking.

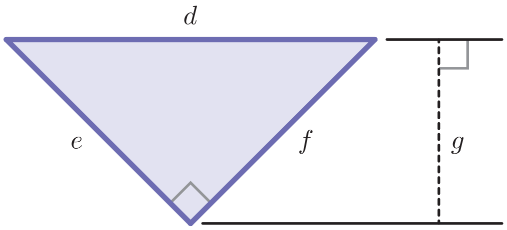
   b. Did Clare decompose the trapezoid into two identical shapes? Explain your thinking.

Name: ______________ Date: ______________ Period: ______________

3. Determine the area of the trapezoid. Each box is 1 square unit.

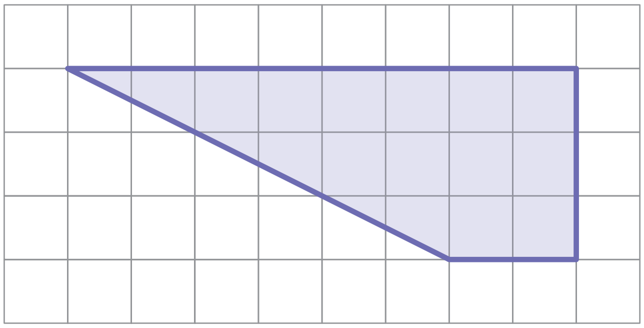


4. Can side d be a base of the triangle shown? If so, which length would be the corresponding height? If not, explain why not.

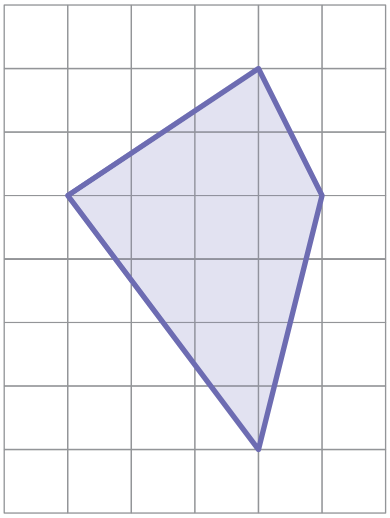


5. Decompose each shape into triangles.

a

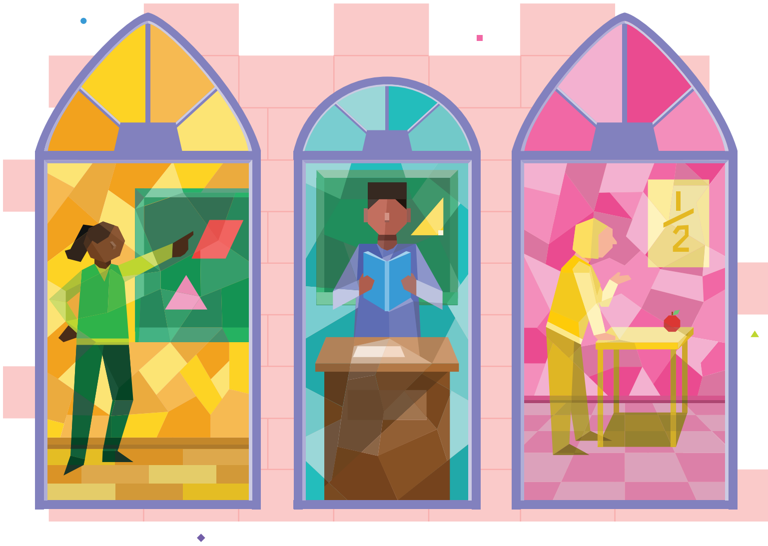


b



Polygons

Let's investigate polygons and their areas.



Warm-up What Are Polygons?

You will be shown some examples and non-examples of polygons, as well as ten other figures, labeled L–U. Classify each figure as an example or a non-example of a polygon and record the figure letters in the appropriate section of the graphic organizer shown. Then describe the characteristics of a polygon based on features of the examples.

Examples:	Non-examples:
Characteristics:	Definition:

Polygons

Log in to Amplify Math to complete this lesson online.

Name: ________________ Date: ________ Period: ________

Activity 1 Stained Glass

You will be given a ruler and a calculator. You will create a design for a stained glass window that is composed of $8–12$ polygons, or panels, whose areas can all be determined. In the table, sketch each panel from your design and show how its area is calculated. You may use either inches or centimeters.

Polygon	Area

Activity 1 Stained Glass (continued)

Polygon	Area

Are you ready for more?

Determine the sum of the areas of your polygons. The total area of your polygons should equal the area of the piece of paper. Check your work to see if this holds true and explain why it is true.

Note: U.S. Letter size paper has dimensions of $8\frac{1}{2}$ in. by 11 in.

STOP

Name: ________ Date: ________ Period: ________

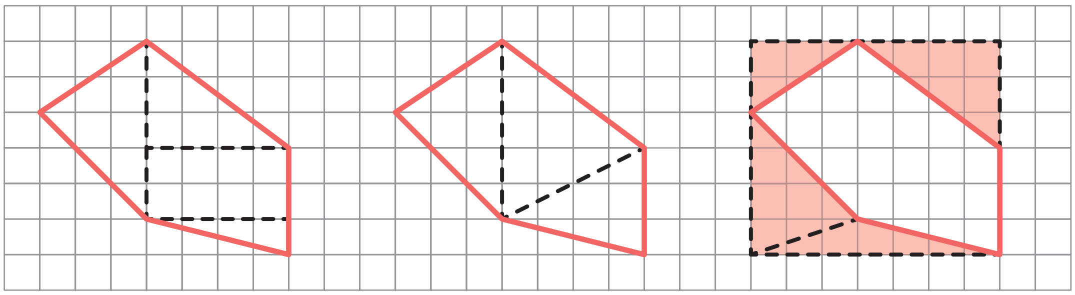
Summary

In today's lesson . . .

You worked with polygons, some familiar with known names and others that are similar in some ways, but different in other ways. A ***polygon*** is a two-dimensional shape composed of sides that are all line segments. For polygons:

- Line segments are straight, not curved.
- Each endpoint of every side connects to an endpoint of exactly one other side.
- The line segments of polygons do not cross each other. A point where two sides intersect is a *vertex* of the *polygon*. **Note:** The plural of *vertex* is *vertices*.
- There are always an equal number of vertices and sides for any polygon. For example, a polygon with 5 sides will have 5 vertices.

You can determine the area of *any* polygon using many familiar strategies, such as decomposing, rearranging, composing, or enclosing, to form shapes with known and well-defined areas — namely triangles and parallelograms. These areas can be determined using a grid or by using formulas, as long as the necessary lengths are known or can be determined, such as by measuring with a ruler.

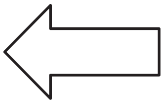


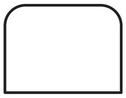
Reflect:

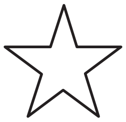
Practice

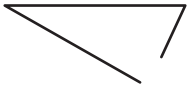
Name: ________________ Date: ________ Period: ________

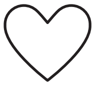
1. Select *all* the polygons.

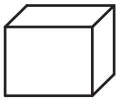
A. 

B. 

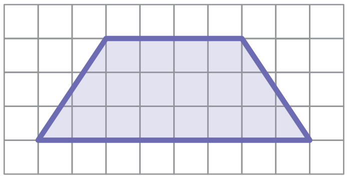
C. 

D. 

E. 

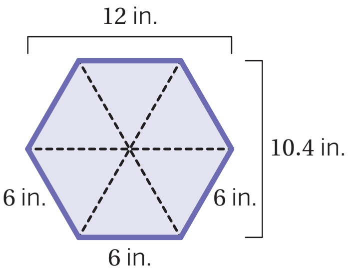
F. 

2. Determine the area of the trapezoid shown. Explain or show your strategy. Each square has an area of 1 square unit.

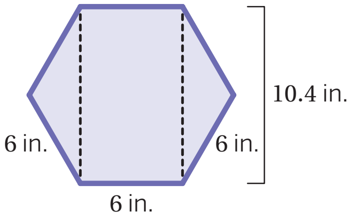


3. Lin and Andre used different methods to determine the area of the hexagon shown, where each side measures 6 in.

- Lin decomposed the hexagon into six identical, equilateral triangles.
- Andre decomposed the hexagon into a rectangle and two triangles.



Lin's method

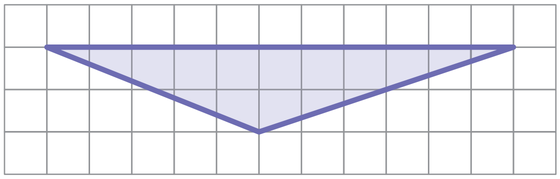


Andre's method

Show the calculations each person could have used to determine the area of the hexagon.

Name: .... Date: .... Period: ....

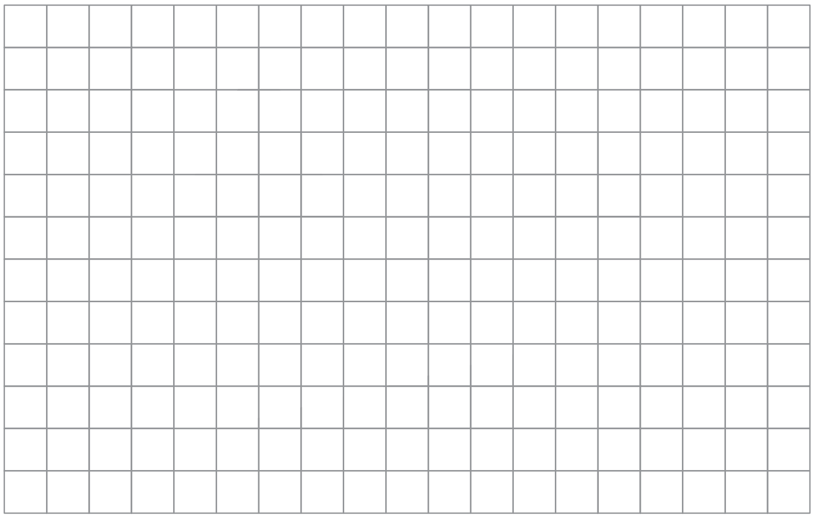
4. Identify a base and a corresponding height that can be used to determine the area of the triangle shown.



a. Label the base b and the corresponding height h.

b. Calculate the area of the triangle. Show your thinking.

5. On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



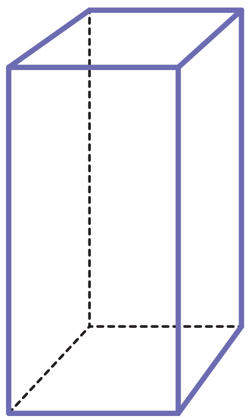
6. Consider Figure G.

a. What makes this figure three-dimensional?

b. How many polygons do you see along the surface of this prism? What type of polygons are they?

c. What type of three-dimensional solid is this figure?

Figure G



My Notes:

How did a misplaced ruler change the way you shop?

Not many people remember Robert Gair. But back in his day, he was a wealthy industrialist. In 1853, he came to New York from Scotland at the age of 14. And when the Civil War broke out just a few years later, he joined the Union army, earning the rank of captain before returning to New York with $10,000 to start a business manufacturing square-bottomed paper bags.

Then, on one fateful day in 1879, a ruler in one of Gair's bag-folding machines slipped out of place. Rather than folding the bags, it sliced through about 20,000 sheets of inventory, ruining them.

But where others might have simply seen destroyed inventory, Gair saw opportunity. It turned out that the slicing made the bags easier to fold up into containers.

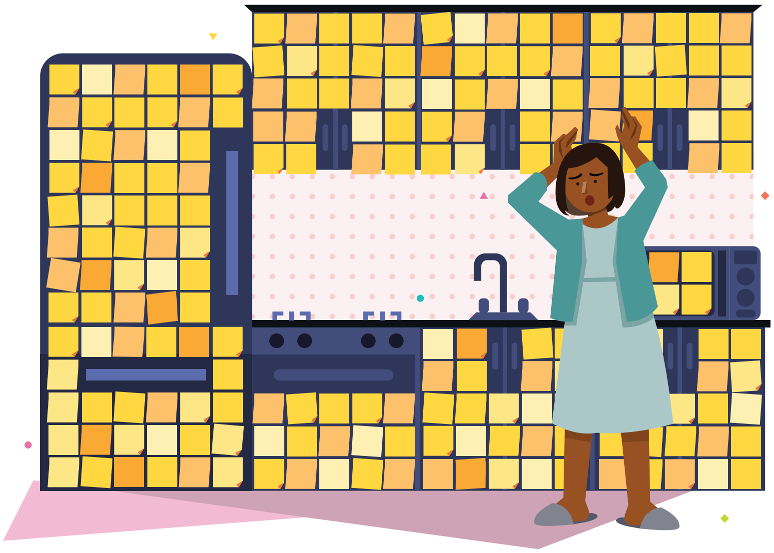
And so the cardboard box was born!

That might not seem like a big deal to us in the 21st century, but cardboard boxes changed how we packaged things forever. Almost 150 years later, cardboard boxes are everywhere: in our kitchens, closets, and mailboxes. And practically everything that gets delivered these days comes in a cardboard box.

Gair's genius lay in his ability to see how the surface area of a simple flat figure could be manipulated to cover a three-dimensional space. It's time for you to think like Gair and take area into the third dimension.

What Is Surface Area?

Let's cover the surfaces of some three-dimensional objects.



Warm-up Notice and Wonder

In the next activity, you will watch a video of a cabinet being covered with sticky notes. Consider these images of moments captured from that video. What do you notice? What do you wonder?

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Covering the Cabinet

Plan ahead: In what ways can you show others respect when discussing different strategies used to complete the task?

Part 1

1. What information would you need to know in order to determine the total number of sticky notes it would take to cover the entire cabinet pictured in the Warm-up?

Part 2: Watch the first part of a video of the cabinet being covered with sticky notes.

2. Use the information from the video to calculate the total number of sticky notes needed to cover the cabinet entirely. Show or explain your thinking.

Are you ready for more?

How many sticky notes are needed to cover the outside of 3 cabinets pushed together (including the bottom of each cabinet)? Would the total number of sticky notes needed change if the cabinets were pushed together in different ways?

Activity 2 Building With Unit Cubes

You will be given 12 unit cubes. Each face of the cubes has an area of 1 square unit.

1. Use all 12 cubes to build a rectangular prism. Record the following dimensions of your prism.

   a. Length:

   b. Width:

   c. Height:

2. For your prism, determine each of the following. Show or explain your thinking.

   a. Volume, in cubic units:

   b. Surface area, in square units:

Are you ready for more?

Imagine you connected your rectangular prism to your partner's prism to form a new figure.

1. Would this new figure always be another rectangular prism?

2. If you started with identical prisms, would the total surface area of the new figure always be twice the surface area of your prism? Explain your thinking.

STOP

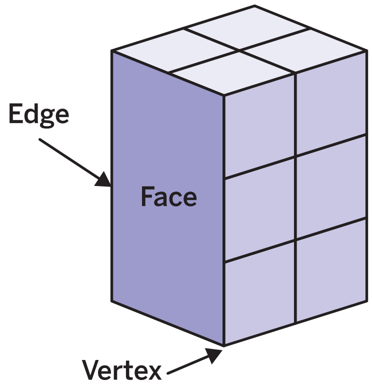
Name: ______________________ Date: ____________ Period: ____________

Summary

In today's lesson . . .

You explored some different attributes of a special type of three-dimensional solid, a *rectangular prism*, which is composed of six rectangular *faces*.

- A ***face*** of a three-dimensional solid is any one of the two-dimensional shapes that are joined to make the solid's outer surface.
- A shared side of two faces is called an ***edge***.
- The intersection point of two (or more) edges is called a ***vertex***.



In a rectangular prism, there will always be three pairs of identical (opposite) faces. Sometimes, two or more of the faces are identical. For example, in a cube, all six faces are identical squares.

Volume and surface area are two measurable attributes of all three-dimensional solids.

- ***Volume*** measures the number of unit cubes that can be packed into a figure without gaps or overlaps. Because volume is a *three-dimensional measure*, volume is expressed in *cubic units*.
- ***Surface area*** is the number of unit squares it takes to cover all of the faces of a solid without gaps or overlaps. Because surface area is a *two-dimensional measure*, it is expressed in *square units*. The surface area for any three-dimensional solid is equal to the total area (i.e., the sum of the areas) of all the individual faces.

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Which statement describes the surface area of this trunk?

Ttatty/Shutterstock.com

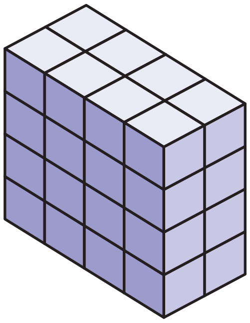
   A. The number of square inches that cover the top of the trunk.

   B. The number of square feet that cover all the outside faces of the trunk.

   C. The number of square inches of horizontal surface inside the trunk.

   D. The number of cubic feet that can be packed inside the trunk.

2. This rectangular prism is 4 units high, 4 units wide, and 2 units long. What is its surface area?



   A. 16 square units

   B. 32 square units

   C. 48 square units

   D. 64 square units

3. Compare the surface areas of Figure A and Figure B. Show or explain your thinking.

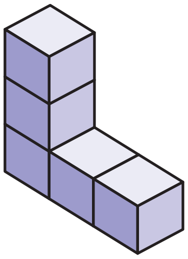


Figure A

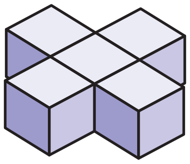
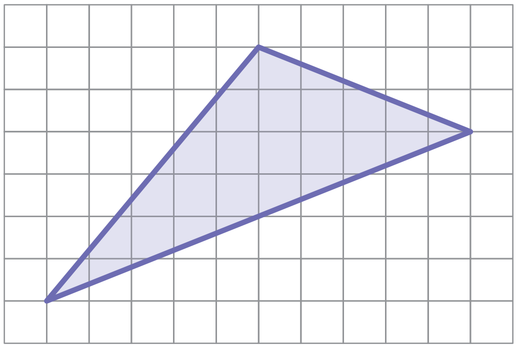


Figure B

4. Determine the area, in square units, of the shaded region. Show or explain your thinking.

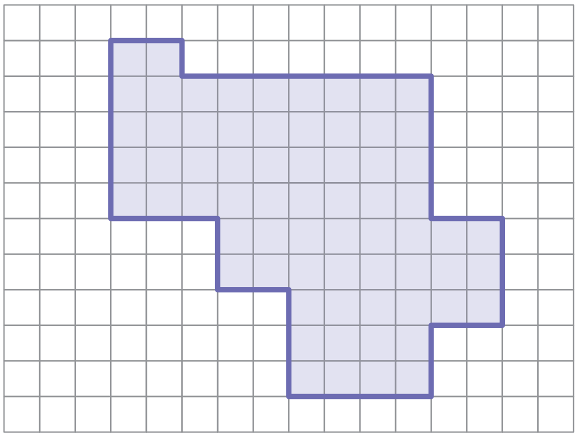


5. Determine the stated measure for each shape described.

a. The length of the base of a parallelogram is 12 m and its corresponding height is 1.5 m. What is the area of the parallelogram?

b. The length of the base of a triangle is 16 in. and its corresponding height is $\frac{1}{8}$ in. What is the area of the triangle?

6. Determine the area of the shaded figure. Show your thinking.



Unit 1 | Lesson 15

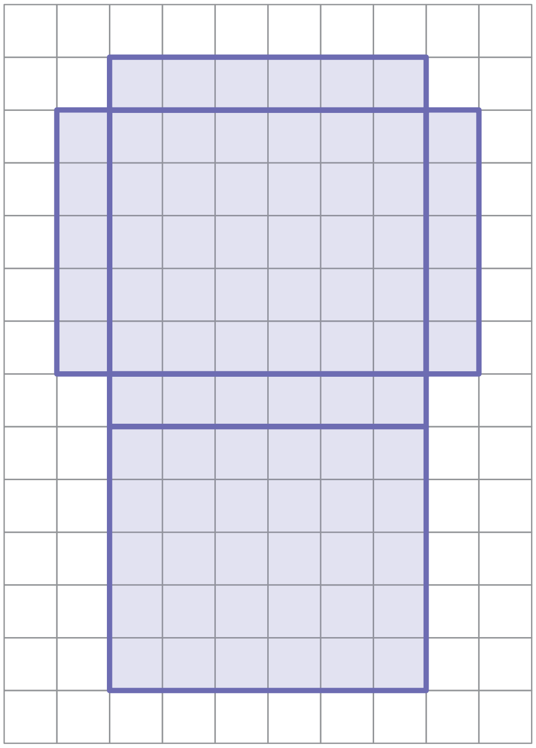
Nets and Surface Area of Rectangular Prisms

Let's use nets to calculate the surface area of rectangular prisms.

Warm-up "Unfolding" a Rectangular Prism

You will watch a video of a rectangular prism being "unfolded."

Here is a two-dimensional representation of the "unfolded" rectangular prism. Label the top, bottom, left, right, front, and back faces of the original three-dimensional figure.



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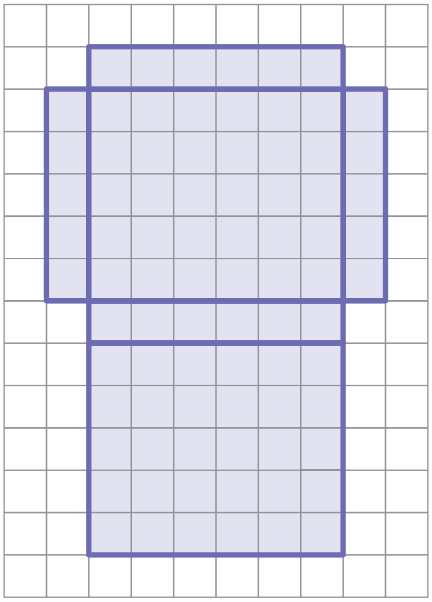
Name: .... Date: .... Period: ....

Activity 1 Using the Net of a Rectangular Prism

Here is the same net of the rectangular prism from the Warm-up.



1. Calculate the surface area of the rectangular prism in square units. Show or explain your thinking.

2. Determine whether each figure is a net of a rectangular prism. Be prepared to explain your thinking.

Figure A

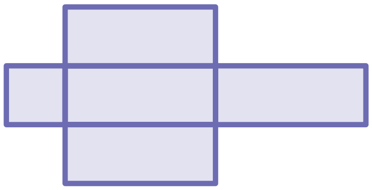


Figure B

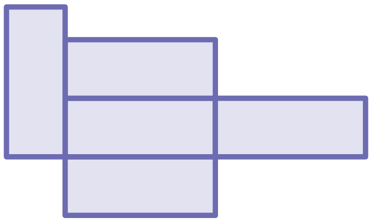


Figure C

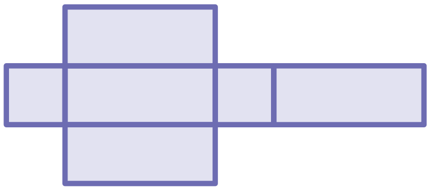
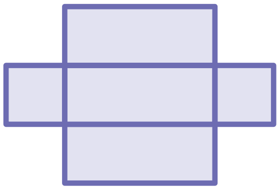
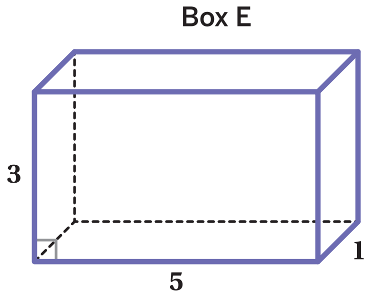


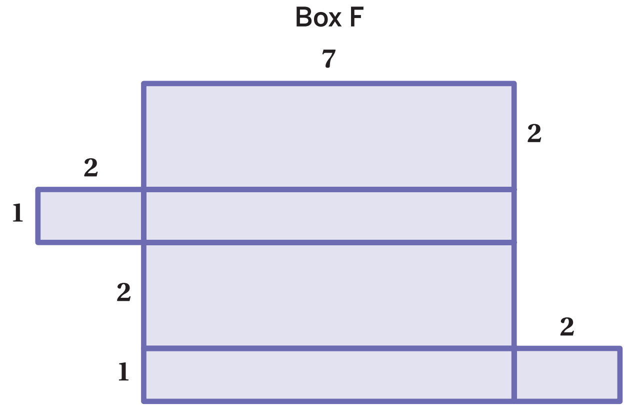
Figure D



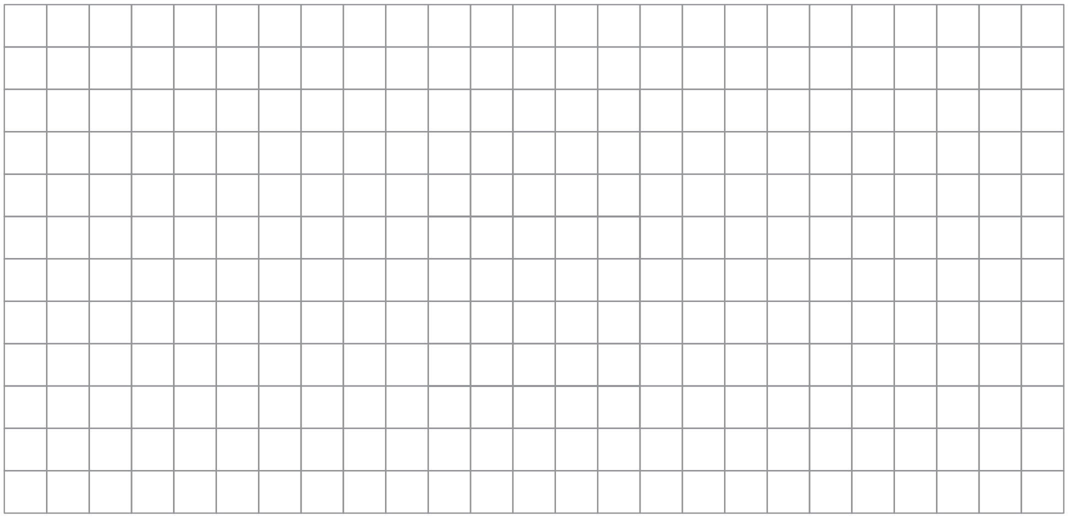
Activity 2 Comparing Boxes

Here is a model of one box and a net of another box. All lengths are in inches.





1. Draw a net for Box E on the grid.



2. Which box uses the least cardboard?

3. If each box was packed with 1-in. unit cubes, which box would be packed with more cubes? Show or explain your thinking.

STOP

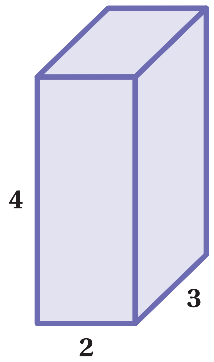
Name: .... Date: .... Period: ....

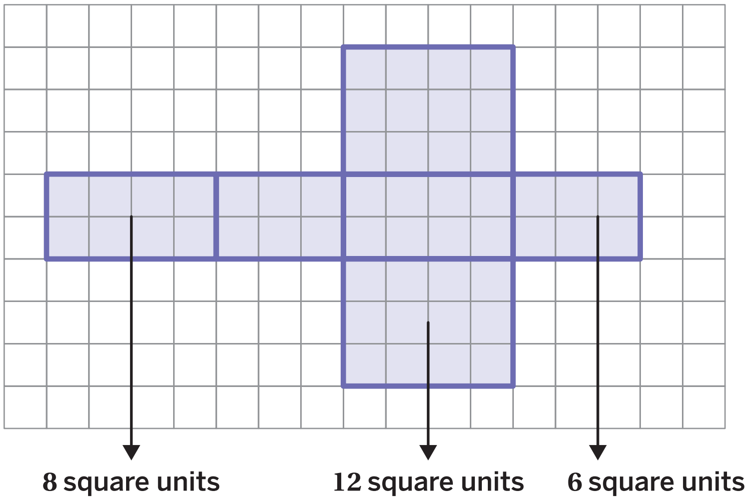
Summary

In today's lesson . . .

You saw that a ***net*** is a two-dimensional representation of a three-dimensional solid that shows the result of "unfolding" the solid such that all of the faces are clearly visible. A net can also be cut and folded to form a three-dimensional model of its corresponding solid.

Because nets show all of the polygons that form the faces of the solid, they are useful for calculating the solid's surface area. For example, the net of this rectangular prism shows three pairs of identical rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units. The surface area of the rectangular prism is 52 square units because the sum of the areas of all the faces is $8 + 8 + 6 + 6 + 12 + 12 = 52$.



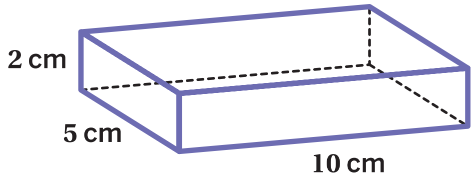


Reflect:

Practice

Name: .... Date: .... Period: ....

1. Refer to the rectangular prism shown.



a. Use the grid to draw a net for the prism. The length of one grid square is 1 cm. Label the top, bottom, left, right, front, and back faces.

b. Determine the surface area of the prism.

2. A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

Practice

3. Prism A and Prism B are both rectangular prisms. Prism A's dimensions are 3 in. by 2 in. by 1 in. Prism B's dimensions are 1 in. by 1 in. by 6 in. Select *all* of the statements that are true.

   A. Prisms A and B have the same number of faces.

   B. More 1-in. cubes can be packed into Prism A than into Prism B.

   C. Prisms A and B have the same surface area.

   D. The surface area of Prism B is greater than that of Prism A.

4. Select *all* of the units that would be appropriate to describe surface area.

   A. Square meters

   B. Feet

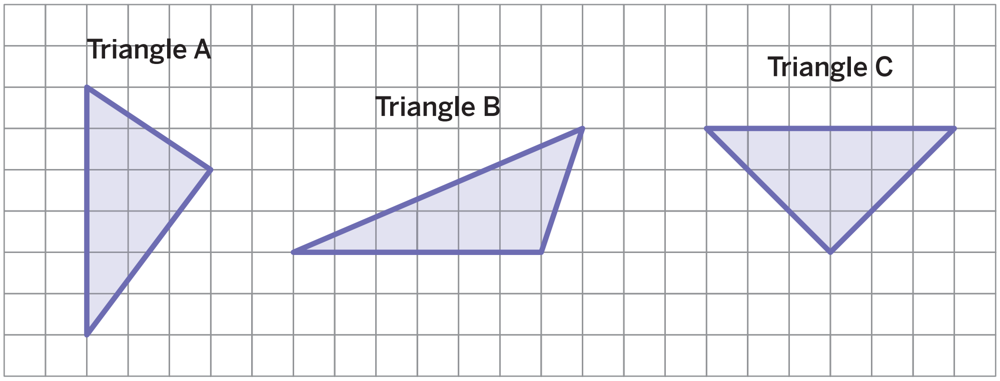
   C. Centimeters

   D. Cubic inches

   E. Square inches

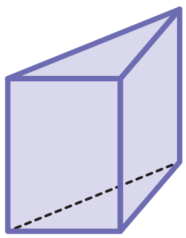
   F. Square feet

5. Show how you know that each of these triangles has an area of 9 square units.

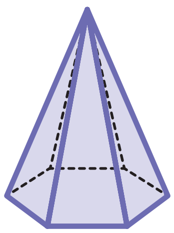


6. Name *all* the polygons that make up the faces of these three-dimensional figures.

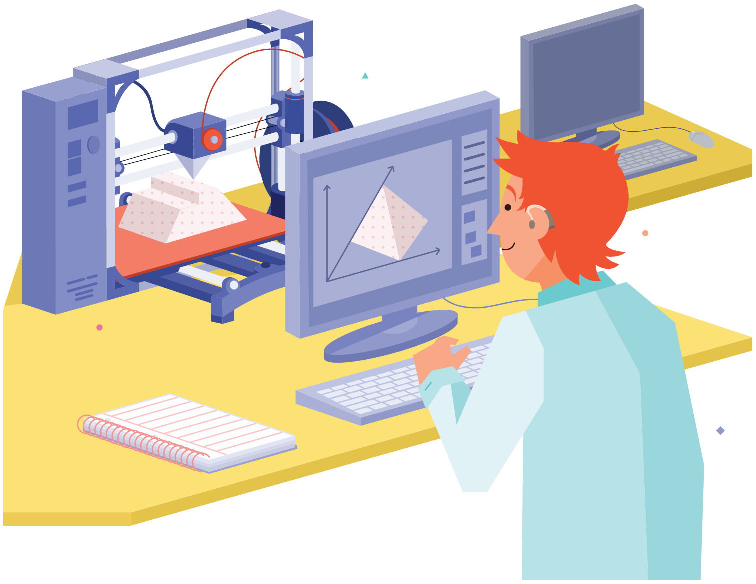
a



b



Nets and Surface Area of Prisms and Pyramids

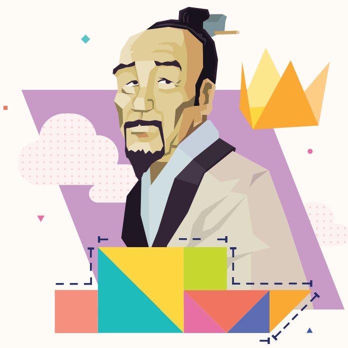


Let's use nets to calculate the surface areas of other polyhedra.

Warm-up Card Sort: Three-Dimensional Figures

Third-century mathematician Liu Hui discovered and proved many relationships among three-dimensional solids with shared features and dimensions. You will be given a set of cards that show three-dimensional figures. Sort the figures into different groups of your choosing, and explain your thinking.

Featured Mathematician



Liu Hui

Liu Hui was born about 225 C.E. near what is now Zibo, China. He is most noted for writing commentaries on problems addressing number theory, geometry, algebra, and trigonometry, in the ancient text called "Nine Chapters on the Mathematical Art."

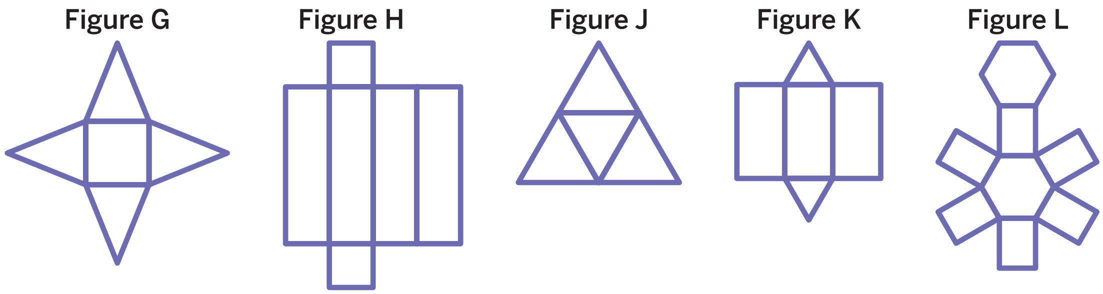
One idea presented there, possibly for the first time anywhere, is known as Liu Hui's Cube Puzzle. It shows how a cube can be decomposed into three solids with volumes of exactly $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ the volume of the cube.

Log in to Amplify Math to complete this lesson online.

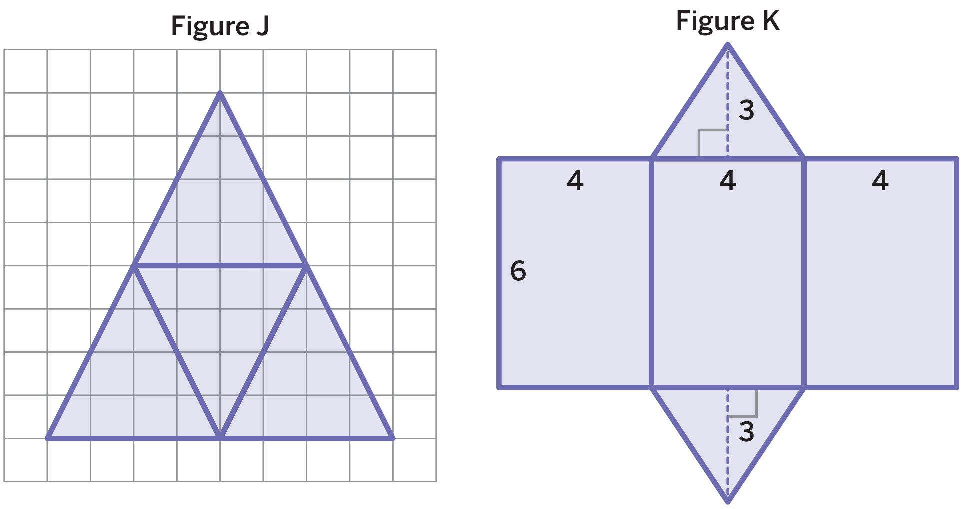
Name: ______ Date: ______ Period: ______

Activity 1 Using Nets to Calculate Surface Area

1. Nets of five polyhedra are shown. Which are prisms and which are pyramids? Be prepared to explain your thinking.



2. The nets for Figures J and K are shown.



a. Label the base(s) of each three-dimensional figure.

b. Name the type of polyhedron that each net would form when assembled.

c. Determine the surface area of each polyhedron. Show your thinking.

Activity 2 Surface Area of Prisms and Pyramids

You will be given a two-dimensional drawing of a polyhedron measured in units.

1. Draw a net for your polyhedron on the grid. Each grid square represents 1 unit.

2. Calculate the surface area of your polyhedron. Show your thinking.

Historical Moment

Volume of an Incomplete Pyramid

The Rhind Mathematical Papyrus (~1650 B.C.E.) is one of the most famous surviving examples of ancient Egyptian mathematics. But an even older document, the Moscow Mathematical Papyrus (~1850 B.C.E.), has survived. Both works present a bunch of math problems and solutions — or in some cases, what were thought to be solutions at the time.

One example in the Moscow Mathematical Papyrus is how to determine the volume of a *frustum* — a pyramid with the top chopped off, or in other words an “incomplete” pyramid. For a pyramid with a square base that has area a, and where the top of the incomplete pyramid at height h is also a square with area b, the formula is: $\frac{1}{3} \cdot h \cdot (a^2 + ab + b^2)$.

Try drawing a frustum. What would you need to know to determine its surface area?

STOP

Name: ____________________ Date: __________ Period: __________

Summary

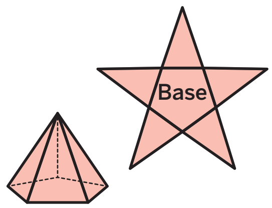
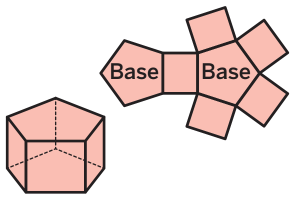
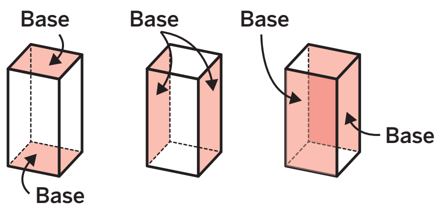
In today's lesson . . .

You worked with two types of solids — prisms and pyramids. Each of these is an example of a closed three-dimensional figure with flat faces that are all polygons, called a ***polyhedron***. (The plural of *polyhedron* is *polyhedra*.) A ***base*** (of a prism or pyramid) is a special face of a polyhedron, defined relative to the type of solid.

A ***pyramid*** has one *base*. All of the other faces are triangles meeting at a single vertex.

A ***prism*** has two *bases*, which are always parallel, identical copies of some polygon. All of the other faces are parallelograms (often rectangles). Because a rectangular prism has three pairs of parallel and identical rectangular faces, any of these pairs can represent the bases.

Both pyramids and prisms are named according to the shape of their bases.

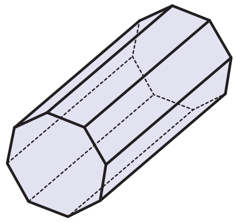
<table>
<tr><th>Pentagonal pyramid</th><th>Pentagonal prism</th><th>Rectangular prism</th></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
</table>

The surface area of a polyhedron is the sum of the areas of all its faces. Because a net shows every face of a polyhedron at once, it can be helpful in calculating surface area.

Reflect:

Name: .... Date: .... Period: ....

1. Refer to the polyhedron shown.



a Explain how you know the figure is a polyhedron.

b Is this polyhedron a prism, a pyramid, or neither? Explain your thinking.

c How many faces, edges, and vertices does it have?

2. Refer to each polyhedron and its corresponding net. Label all of the edges in each net with the correct lengths.

a **Figure A**

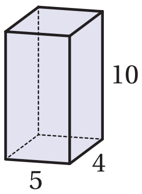
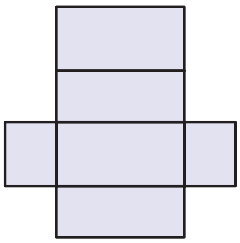


Figure A



b **Figure B**

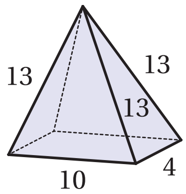
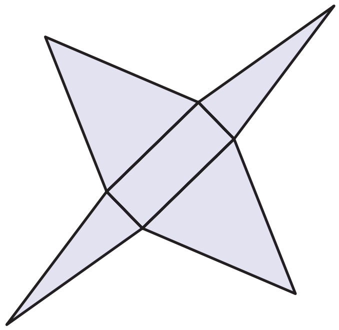
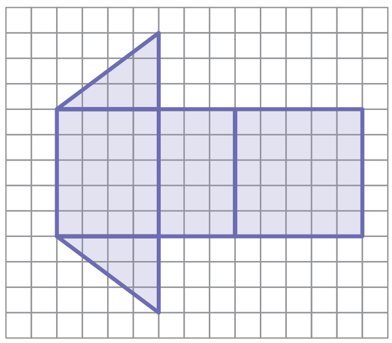


Figure B



3. Refer to the net.



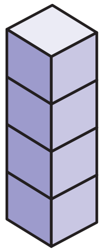
a Name the type of polyhedron that can be assembled from this net. Explain your thinking.

b Determine the surface area of this polyhedron. Show your thinking.

Name: .... Date: .... Period: ....

Practice

4. The figure shown is a representation of a rectangular prism built from unit cubes.

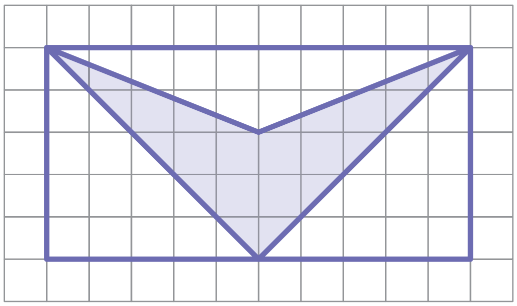


a Determine the volume in cubic units.

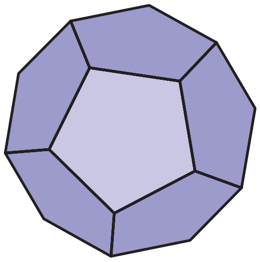
b Determine the surface area in square units.

c Determine whether the following statement is *true* or *false*. Show or explain your thinking. If you double the number of cubes and stack them all in the same way, *both* the volume and surface area will double.

5. Determine the area of the shaded figure shown. Show your thinking.



6. This image shows exactly half of a polyhedron called a *dodecahedron*. What polygons make up the faces of a dodecahedron? How many faces does it have?



Constructing a Rhombicuboctahedron

Let's use nets to construct a rhombicuboctahedron.

Warm-up Notice and Wonder

Consider these images of the National Library of Belarus. What do you notice? What do you wonder?

webhobbit/Shutterstock.com

Grisha Bruev/Shutterstock.com

1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Constructing a Model of the Library

The National Library of Belarus is a *rhombicuboctahedron*, a polyhedron composed of eighteen squares and eight triangles. Each square face has an edge length of 24 m, and each triangular face has a height of approximately 20.8 m.

1. Write an expression to represent the surface area of the National Library of Belarus. Then evaluate your expression to determine its surface area, in square meters.

2. You will be given a copy of a net for the library, a pair of scissors, and some glue or tape. Use these to assemble a model of the National Library of Belarus.

Are you ready for more?

The exterior of the National Library of Belarus is completely covered with glass windows. The total surface area represents approximately the total amount of glass, in square meters, that is needed to cover the exterior of the library.

1. How well do you think your response to Problem 1 in Activity 1 represents the actual amount of glass that was used to build the library? Do you think it is more likely to be an overestimate or an underestimate? Explain your thinking.

2. Show some calculations that could be used to estimate the difference between your original calculation and the actual surface area covered by glass.

Reflect: How did you remain positive and confident while constructing the model?

STOP

Summary

In today's lesson . . .

You applied key concepts of nets and surface area to create a three-dimensional model of the Belarus National Library, which is a polyhedron called a *rhombicuboctahedron*.

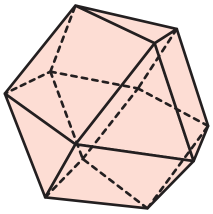
The surface area of any polyhedron is the total area (i.e., the sum of the areas) of all the individual faces. To simplify calculations, you can group faces that are identical copies of one another. For example, because a rhombicuboctahedron is composed of eighteen identical squares and eight identical triangles, you can multiply the area of one square by 18 and the area of one triangle by 8, and then add these areas.

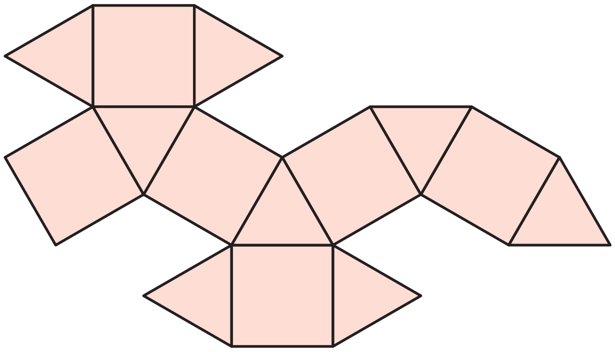
Reflect:

Name: .... Date: .... Period: ....

Practice

Use the figures showing a cuboctahedron and a possible net to complete Problems 1–3.





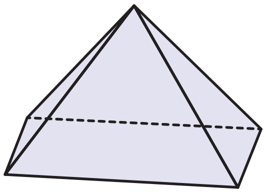
1. How many faces does a cuboctahedron have? How many of the faces are squares? How many of the faces are triangles?

2. Is a cuboctahedron a polyhedron? How do you know?

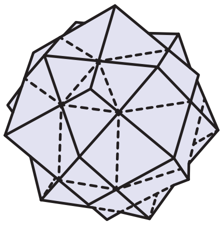
3. Each square face has an edge length of 3 in., and each triangular face has a height of approximately 2.6 in. Calculate the surface area of the cuboctahedron.

4. Select *all* of the figures that are polyhedra.

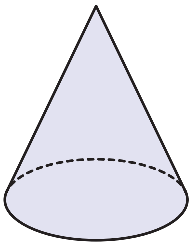
A.



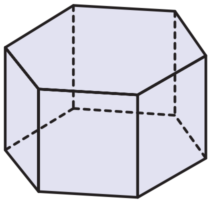
B.



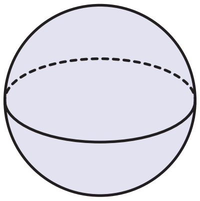
C.



D.



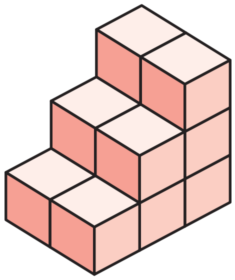
E.



Practice

Name: ________ Date: ________ Period: ________

5. This figure is composed of 12 unit cubes.



a What is the surface area of the figure? Show or explain your thinking.

b How would its surface area change if the top two cubes are removed?

6. Complete the missing value in each row of the table. The first row has been completed for you:

	Power	Expanded	Product
	3^4	$3 \cdot 3 \cdot 3 \cdot 3$	81
a	5^2		
b		$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	
c		$4 \cdot 4 \cdot 4$	
d			36

Name: .... Date: .... Period: ....

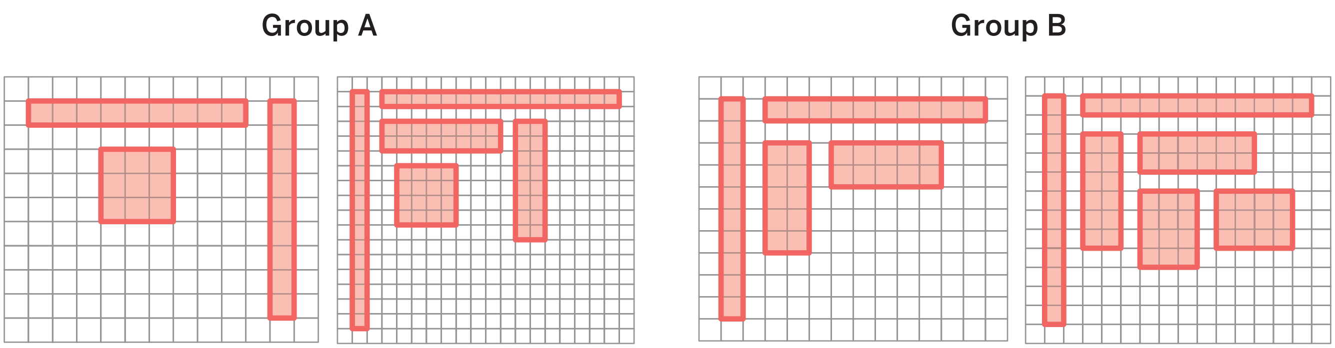
Unit 1 | Lesson 18

Simplifying Expressions for Squares and Cubes

Let's write expressions for the attributes of squares and cubes.

Warm-up How Do They Compare?

Explain how Group A and Group B are similar and how they are different.



1. Group A and Group B are similar because . . .

2. Group A and Group B are different because . . .

Log in to Amplify Math to complete this lesson online.

Activity 1 Building "Perfect" Cubes

You will be given 32 unit cubes.

1. Build the largest cube possible, using any or all of your 32 unit cubes. How many unit cubes did you use?

2. What do you notice about the edge lengths in the cube you built?

3. Determine each of the following for your cube. Show your thinking and include appropriate units.

   a. Area of each face:

   b. Surface area:

   c. Volume:

4. Could you build a cube using exactly 20 unit cubes? Explain your thinking.

5. How many different-sized cubes are possible if you can use *up* to 32 unit cubes for each cube?

Are you ready for more?

Imagine you teamed up with another group and now have 64 unit cubes to use to build cubes.

1. What is the largest cube you could make? Explain your thinking.

2. Calculate the volume and surface area of the perfect cube you described in Problem 1.

   a. Volume:

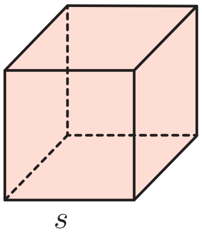
   b. Surface Area:

3. How many groups would you need to team up with in order to have enough unit cubes to build a cube with a height of 10 units? Assume each group has 64 unit cubes.

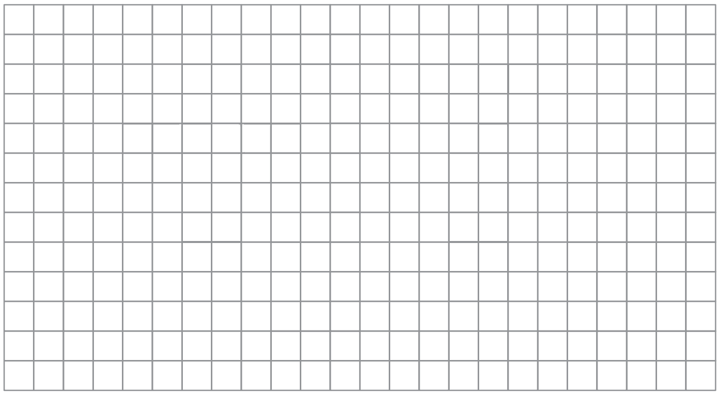
Name: ________________ Date: ________ Period: ________

Activity 2 Writing Expressions for the Attributes of Cubes

Consider the cube with edge length s.



1. Draw a net of the cube.



2. Write an expression to represent each of the following for a cube with side length s. Include the appropriate units.

   a. Area of each face:

   b. Surface area:

   c. Volume:

Are you ready for more?

The number $15{,}625$ is a *perfect square* because it is equal to $125 \cdot 125$. It is also a *perfect cube* because it is equal to $25 \cdot 25 \cdot 25$. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

STOP

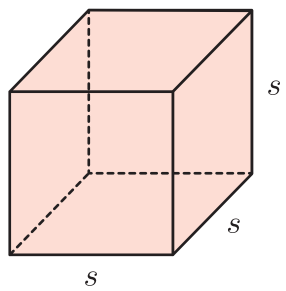
Summary

In today's lesson . . .

You explored perfect squares and perfect cubes. A ***perfect square*** is the product of a factor and itself. The number 16 is a perfect square because $4 \cdot 4 = 16$. A ***perfect cube*** is the product of a factor multiplied by itself three times. The number 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 27$.

A perfect square can be represented geometrically as the area of a square with whole number side lengths because its sides are all identical copies of one another. A perfect cube can be represented geometrically as the volume of a cube with whole number edge lengths because its faces are all identical squares.

Consider the cube with edge length s units. When you substitute s into the known formulas for area of a parallelogram (a face), and surface area and volume of rectangular prisms, the resulting expressions can be simplified. And those simplified expressions can be used to make calculations more efficient when working with cubes.



- **Area:** The area of each square face is equal to $s \cdot s$ square units.
- **Surface area:** The sum of the areas of all six faces, $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$, or $6 \cdot (s \cdot s)$ square units.
- **Volume:** The volume is equal to $s \cdot s \cdot s$ cubic units.

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. Decide whether each number is a perfect square, a perfect cube, *both* a perfect square and a perfect cube, or *neither*. Show or explain your thinking.

a. 1

b. 3

c. 8

d. 16

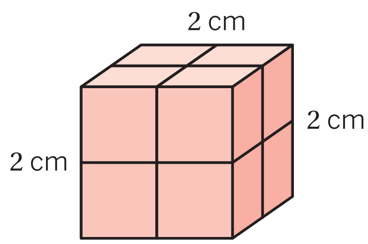
e. 20

f. 64

g. 100

h. 1,000

2. For this cube, calculate each of the following measurements. Be sure to include appropriate units.



a. Area of each face:

b. Surface area:

c. Volume:

Practice

Name: .... Date: .... Period: ....

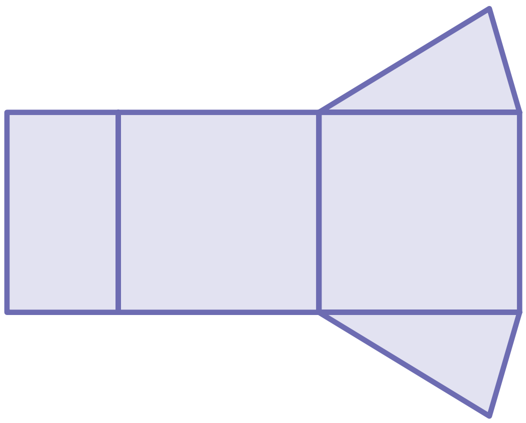
3. Determine the stated measure(s) in each scenario, using appropriate units.

a A square has side length 4 cm. What is its area?

b The area of a square is 49 square meters. What is its side length?

c The edge length of a cube is 3 in. What is its volume? surface area?

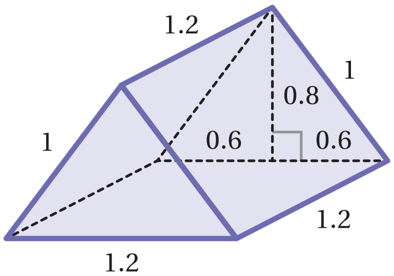
4. Refer to the net.



a What type of polyhedron can be assembled from this net?

b Label the dimensions or measures you would need to know in order to calculate the surface area.

5. Calculate the surface area of the triangular prism shown. All measurements are in meters.



6. Evaluate each expression.

a $10 + 10$

b $10 \cdot 10$

c 10^2

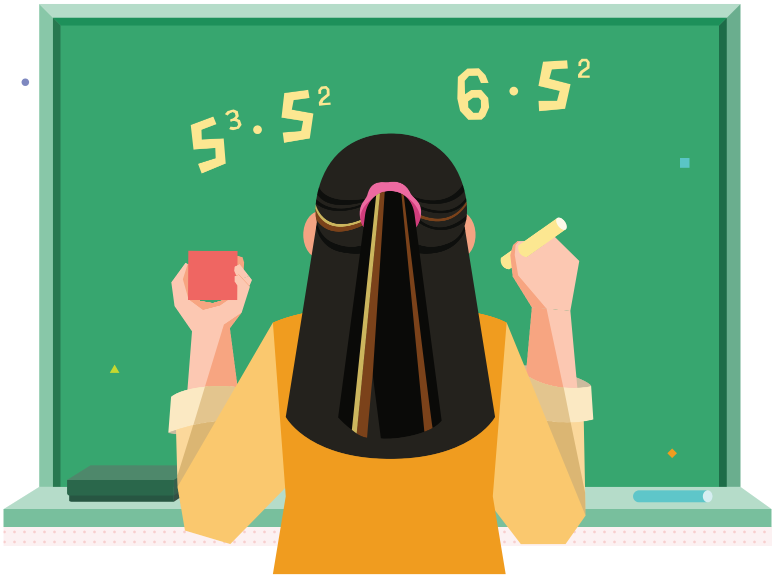
d 10^3

Name: .... Date: .... Period: ....

Unit 1 | Lesson 19

Simplifying Expressions Even More Using Exponents

Let's write expressions with exponents to represent the volume and surface area of cubes.



Warm-up Which Is Greater?

Without calculating the value of each expression, use what you know about operations to determine which expression in each pair represents the greater value. Be prepared to explain your thinking.

1. **Expression A:** $10 \cdot 3$ **Expression B:** 10^3

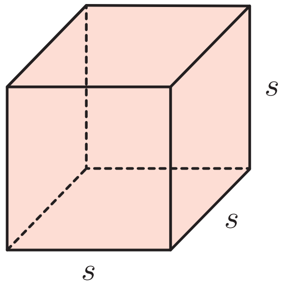
2. **Expression A:** 10^2 **Expression B:** $9 \cdot 9$

3. **Expression A:** $10 + 10 + 10 + 10 + 10 + 10$ **Expression B:** $5 \cdot 10$

Log in to Amplify Math to complete this lesson online.

Activity 1 Card Sort: Sorting Expressions and Units

You will be given a set of cards that contain expressions and related units for this cube with side length s cm.



1. Sort the cards into four groups, expressions, and units that represent:
   - The area of each face
   - The surface area
   - The volume
   - None of these

   Use the table to record how you sorted the cards.

Area of each face	Surface area	Volume	None

2. Explain why the cards in the *None* group did not belong in any other group.

Name: .... Date: .... Period: ....

Activity 2 Using Exponents to Express Attributes of Cubes

1. A cube has an edge length of 7 in. Determine whether each statement is *true* or *false*. Explain your thinking.

   a. The area of each face is 14 in^2 because $7^2 = 14$.

   b. You can calculate the volume of the cube by evaluating 7^3.

   c. The surface area is 84 in^2 because $6 \cdot (7 \cdot 2) = 84$.

   d. You can use in^3 as units to represent volume.

2. A cube has a volume of 125 cubic units. What is its surface area? Show or explain your thinking.

Stronger and Clearer: Share your responses with 2–3 partners to get feedback on your clarity and reasoning. After receiving feedback, revise your responses.

Are you ready for more?

1. Can a cube have the same numeric value for both its surface area and volume? Explain your thinking.

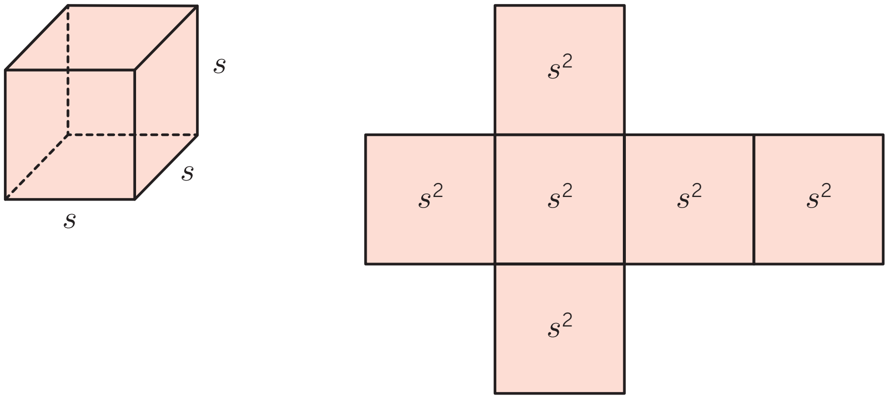
2. Without calculating, how can you determine whether a cube's volume or surface area is greater?

STOP

Summary

In today's lesson . . .

You saw how the formulas for surface area and volume of a cube can be simplified using exponents. Consider a cube with edge length s units and its net.



To calculate the . . .

- *Area of each face*: The expression $s \cdot s$ can be written as s^2. This expression is read as "s ***squared***." The ***exponent*** 2 tells you how many times to multiply the repeated factor s by itself.
- *Surface area*: The expressions $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$ or $6 \cdot (s \cdot s)$ can be written as $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$ or $6 \cdot s^2$.
- *Volume*: The expression $s \cdot s \cdot s$ can be written as s^3. This expression is read as "s ***cubed***."

Exponents are also used to represent the appropriate units for each measurement. For example, if the edge length of a cube was s in., then:

- The area of each face and the surface area would both have units of square inches, which can be written as "in^2."
- The volume would have units of cubic inches, which can be written as "in^3."

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. A cube has an edge length of x cm. Write an expression for each of the following measures of the cube:

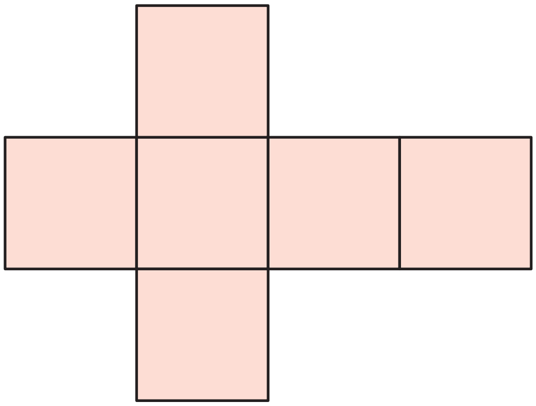
   a. Surface area:

   b. Volume:

2. This net is composed of square faces.

   a. Name the type of polyhedron that can be assembled from this net.

   b. If each square has a side length of 61 cm, write expressions for the surface area and the volume of the polyhedron.



3. Determine each stated measure using appropriate units. Show your thinking.

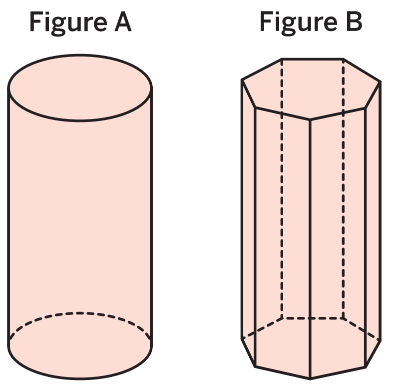
   a. The surface area of a cube with edge length 8 in.

   b. The volume of a cube with edge length $\frac{1}{3}$ cm.

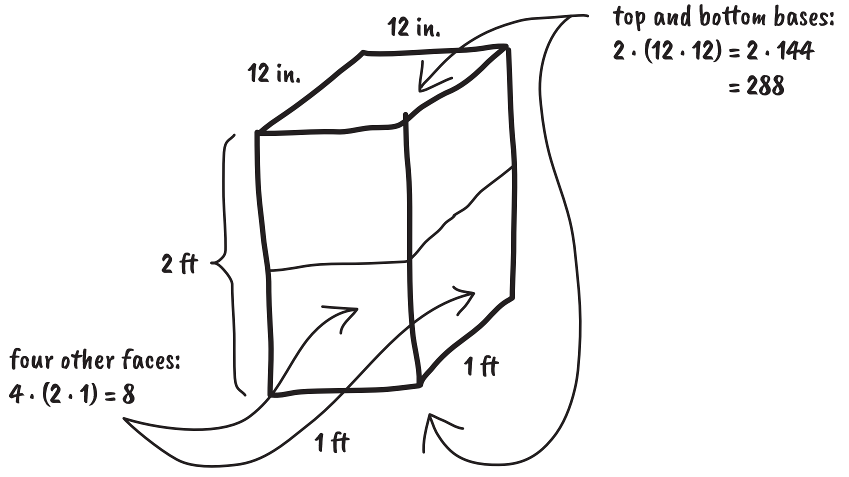
   c. The edge length of a cube that has a volume of 8 ft^3.

Name: .... Date: .... Period: ....

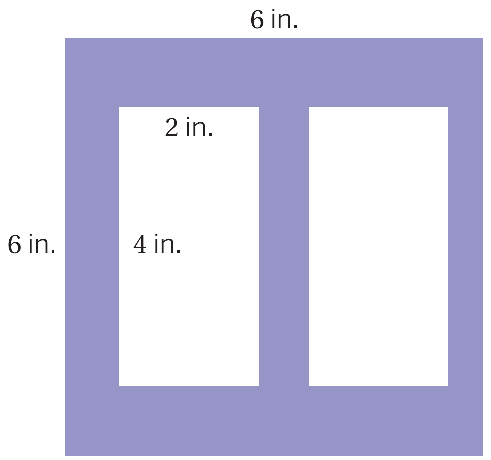
4. Refer to Figures A and B. State whether each figure is a polyhedron. Explain your thinking.



5. Here is Elena's work for calculating the surface area of a rectangular prism with dimensions 1 ft by 1 ft by 2 ft. She concluded that the surface area of the prism is 296 ft^2. Do you agree or disagree? Show or explain your thinking.



6. Determine the area of the shaded region. The unshaded rectangles are identical.



Name: ______ Date: ______ Period: ______

Unit 1 | Lesson 20 – Capstone

Designing a Suspended Tent

Let's design a tent that can hang from trees.

Warm-up Camping Out . . . and Up

Have you ever been camping?

You might know that tents come in a variety of shapes and sizes, but did you know that some can be suspended in trees?

Study these examples of suspended tents.

In your group, discuss:

1. The similarities and differences among these tents.

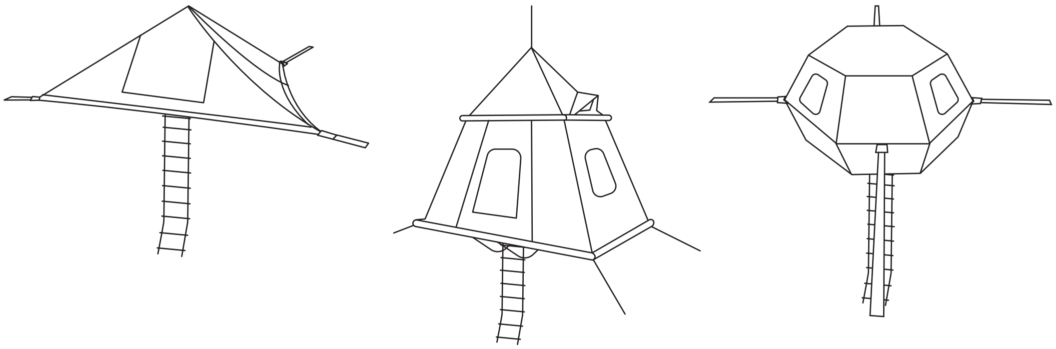
2. The pros and cons of the various designs.

Log in to Amplify Math to complete this lesson online.

Activity 1 Suspended Tent Design

Most tents are made to accommodate adults, but your task is to design a suspended tent to accommodate up to three people that are about your age.

Here are some examples of popular designs.



Develop and sketch a design for your suspended tent. It can look like one of these or can be anything else you come up with. But the tent must include a floor because the ground is not an option! You also need to be able to estimate and justify mathematically the total amount of fabric it will take to construct your tent.

Consider the following specifications to help with your designs.

Height description	Height of tent (ft)	Notes
Sitting height	3	Campers are able to sit, lie, or crawl inside the tent.
Kneeling height	4	Campers are able to kneel inside the tent. Found mainly in 3–4 person tents.
Standing height	5.5	Most campers are able to stand upright.

Sleeping bag measurements for 10–12 year olds:

Name: .... Date: .... Period: ....

Activity 1 Suspended Tent Design (continued)

After the Gallery Tour, discuss the following questions with your group. Record your groups' agreed-upon responses here.

1. Which tent design used the least fabric?

2. Which tent design used the most fabric?

3. Which difference(s) in the designs have the greatest impact on the amount of fabric needed for the tent? Explain your thinking.

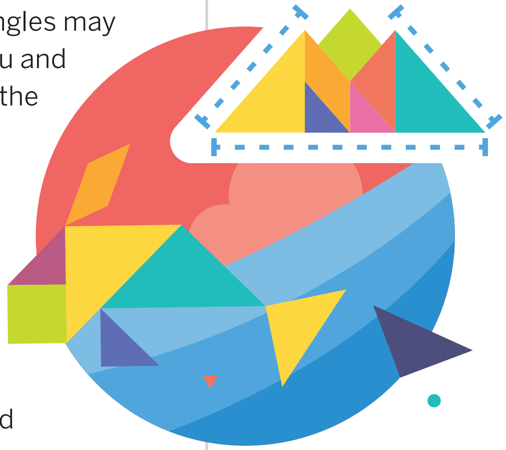
Are you ready for more?

Estimate how much extra floor space would there be if three sleeping bags are placed on the floor, without overlapping. Show and explain your thinking by drawing a sketch of the interior floor space along with your calculations.

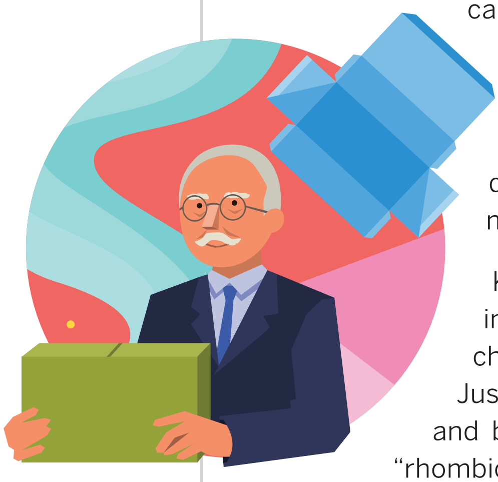
STOP

Unit Summary

Measuring the areas of shapes like squares and rectangles may not seem very complicated. But take a look around you and you'll find that things are a *bit* more complicated than the "perfect" shapes laid out on a sheet of paper.



Take something like the National Library of Belarus, which is shaped like a giant rhombicuboctahedron. Its astounding, gemstone-like structure reflects the treasure of knowledge stored within. And yet, to make this building a reality, architects Viktor Kramarenko and Mikhail Vinogradov needed ways to calculate precisely how much glass and steel they would need.



So where to start? Well, like most complex tasks in life, they can be broken down—or decomposed—into smaller, more manageable parts.

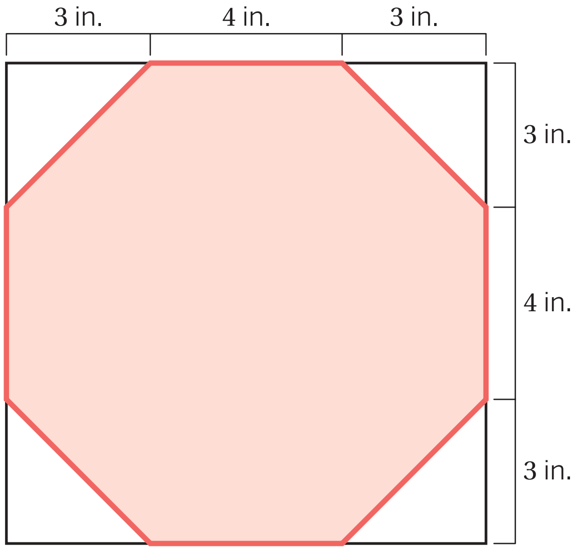
Keep that in mind in the days ahead. Both in and out of math class, you'll face many challenges that might seem overwhelming. Just remember to slow down, take your time, and breathe. Something as scary sounding as a "rhombicuboctahedron" might be nothing more than a few squares and triangles.

See you in Unit 2.

Name: .... Date: .... Period: ....

Practice

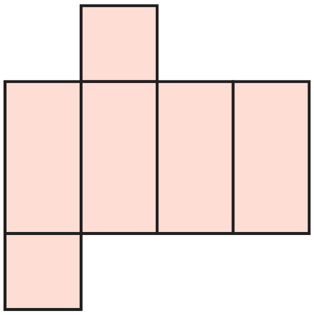
1. Refer to the octagon shown.
Note: The diagonal sides of the octagon are *not* 4 in. long.



a While estimating the area of the octagon, Lin reasoned that it must be less than 100 in^2. Do you agree? Explain your thinking

b Find the exact area of the octagon. Show your thinking.

2. Tyler said that the net shown *cannot* be a net for a square prism because not all the faces are squares. Do you agree with Tyler? Explain your thinking.



Name: .... Date: .... Period: ....

3. Which of these five polyhedra are prisms? Which are pyramids?

Figure A

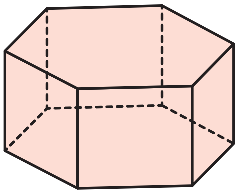


Figure B

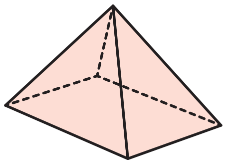


Figure C

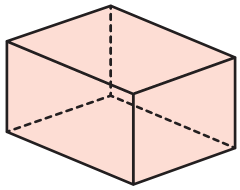


Figure D

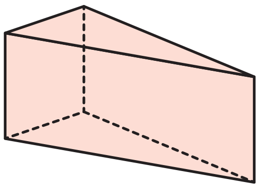
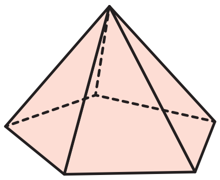


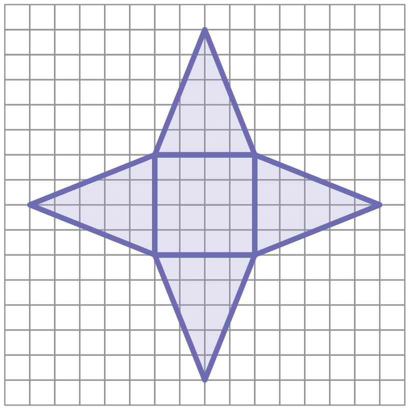
Figure E



Prisms:

Pyramids:

4. Refer to the net shown.



a What three-dimensional figure can be assembled from the net?

b What is the surface area of the figure?
Note: One grid square represents 1 square unit.

5. Match each quantity with an appropriate unit of measurement.

a The surface area of a tissue box	.... square meters
b The amount of soil in a planter box	.... yards
c The area of a parking lot	.... cubic inches
d The length of a soccer field	.... cubic feet
e The volume of a fish tank	.... square centimeters

My Notes: