UNIT 2

Introducing Ratios

A little bit of this and a little bit of that. Well, maybe a lot of that? Wait, I think a ratio can help with this dilemma! Ratios help us see the relationship between one number and another, so when we make guacamole, it doesn't taste awful.

Essential Questions

- What does a ratio say about the relationship between quantities?
- How can ratios reflect fairness?
- How can ratios help you estimate solutions to seemingly impossible real-world problems?
- *(By the way, is it possible to have too much cowbell?)*

SUB-UNIT

1 What are Ratios?

Narrative: Whether it's colors or tiles, having the right amount of each part is the key.

You'll learn . . .
- how ratios represent comparison.
- connections between ratios, multiplication, and division.

SUB-UNIT

2 Equivalent Ratios

Narrative: Ratios can help you keep a rhythm and balance the sounds of music.

You'll learn . . .
- when two ratios are equivalent.
- about common factors and common multiples.

SUB-UNIT

3 Solving Ratio Problems

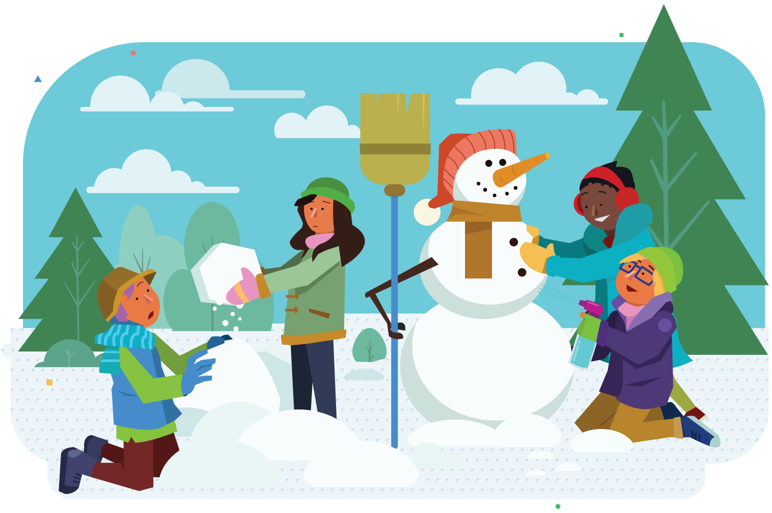
Narrative: Every good cook knows that ratios are an important ingredient of any recipe.

You'll learn . . .
- how to use ratios to find missing values.
- to apply ratios to real-world problems.

The ratio of the different edge lengths of a rectangular prism is 1:2:3. If its volume is 1296 units³, what is the length of its longest edge?

Fermi Problems

Let's explore Fermi problems.



Warm-up Cardiac Rhythm

Describe how you could make a rough estimate to solve this problem: "How many times does your heart beat in a year?" Include any information you would need to know.

Co-craft Questions: Work with your partner to come up with 2–3 questions you might have about the information needed to solve this problem.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 The Fermi Carousel

Enrico Fermi was an Italian scientist born in Rome in 1901. Immediately after receiving the Nobel Prize for Physics in 1938, he and his family immigrated to the United States — "immediately" because Italy's close association with Nazi Germany was unsettling given that Fermi's wife was Jewish. While in the U.S., Fermi became known for his uncanny ability to quickly "guesstimate" solutions to seemingly impossible-to-answer mathematical problems by working with reasonable information and approximations to make back-of-the-envelope calculations. He made a habit of challenging his students and fellow scientists with these types of questions.

The Fermi family arriving in America, Jan. 1939.
AIP Emilio Segrè Visual Archives, Wheeler Collection

Part 1

You and your group will rotate around the room to various stations where Fermi problems have been placed. At each station, you will have a limited amount of time to think about and write down one of three things: assumptions you would have to make, related questions, or approximate answers to any questions from previous groups.

1. How long would it take to read the dictionary?

2. How many balloons could you fit in your classroom?

3. How many hours of television does a 6th grader watch in a year?

4. How long would it take to paddle across the Pacific Ocean?

5. How many liters of water does the school use each week?

6. How many times could you say the alphabet in 24 hours?

7. How many single strands of hair are on your head?

Activity 1 The Fermi Carousel (continued)

8. How many blades of grass are there on a football field?

9. How many grand pianos could you fit in the cafeteria?

10. How many pieces of cooked spaghetti would you need to wrap around the perimeter of your school?

11. How much pudding would it take to fill a swimming pool?

12. If all the books in the school were stacked on top of each other in one pile, how tall would the pile be?

Part 2

You now have one of the Fermi problems to try to solve as a group. Identify the questions, answers, and assumptions that are helpful. Work together to come up with a way you might solve your problem, adding more assumptions and related questions as necessary.

You will be given a separate sheet to illustrate how your group interpreted the information to solve the Fermi problem for others to see. Be sure to include diagrams (or pictures), numbers, and words.

Are you ready for more?

Think about information that would be needed to work through this Fermi problem. Provide a plan for how you would solve the problem.

Some research has shown that it takes 10,000 hours of practice for a person to achieve the highest level of performance in any field — sports, music, art, chess, programming, etc. If you aspire to be a top performer in a field you love, such as Michael Jordan in basketball, Frida Khalo in painting, or Maya Angelou in literature, how many years would it take you to meet that 10,000-hour benchmark if you start now? How old would you be?

STOP

Unit 2 Unit Title

Sensing a Ratio

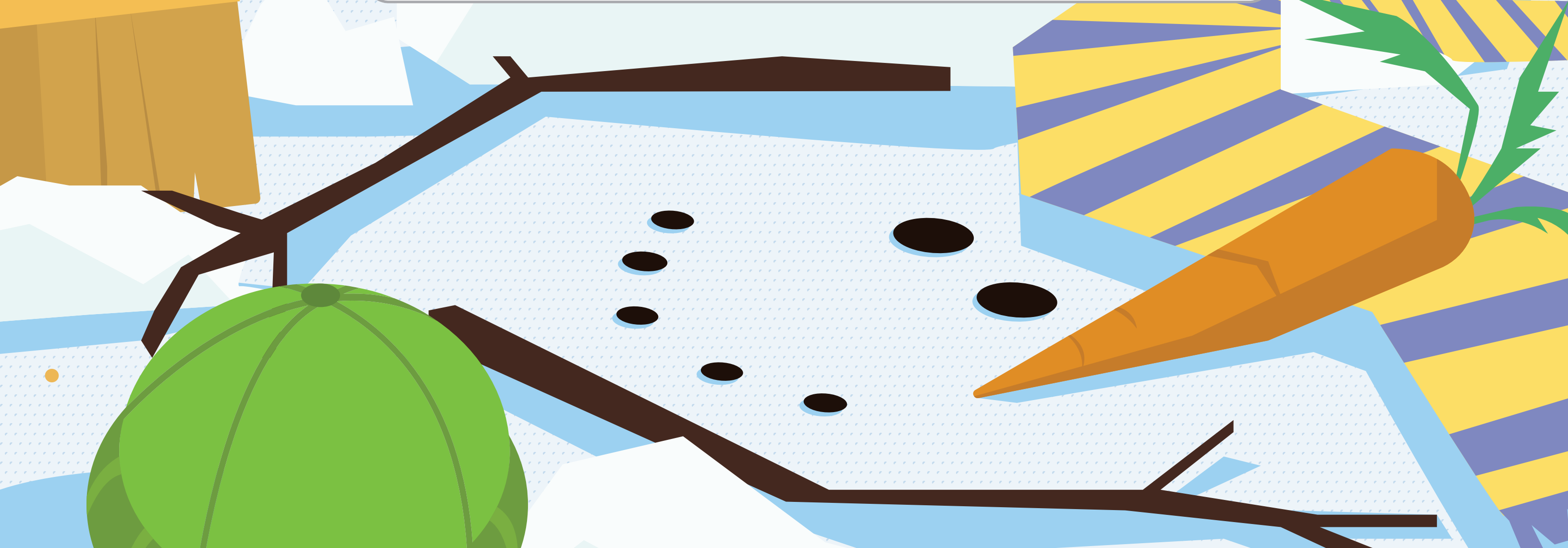
Nothing says winter like a snowman: three massive snowballs for the head and body; twigs for arms; button eyes; and of course, the carrot nose.

The art of building snowmen has been around since the Middle Ages. In the winter of 1511, the peasants of Brussels protested the ruling class by filling their city with 110 snowmen, posed in shocking and embarrassing positions. In 1690, the village of Schenectady, New York posted two snowmen to guard their town, leaving them vulnerable to a raid led by neighboring French Canadians. In 1950, students from the city of Sapporo, Japan built six snow statues in Odori Park. This kicked off what would become Sapporo's world-famous Snow Festival, which today attracts more than 2 million attendees to its massive snow sculpture competition.

It seems like anywhere you can find snow, *someone* has built a snowman nearby. Not all snow is created equal though. Sometimes it's nice and fluffy. But sometimes it's icy and hard, or watery and slushy. For building snowmen, you need snow that's strong and solid, but still easy to shape. Getting that consistency requires the perfect mix of ice and water.

Even with limited experience, you can estimate the right mix by how the snow feels in your hands—how much give it has, the way it holds together. But this relationship between water and ice can also be expressed using numbers, to let you know when it's a good day to build that perfect snowman. And those numbers can even help you estimate whether you have enough snow to make it five feet tall, or five hundred feet tall!

Welcome to Unit 2.



Practice

Name: .... Date: .... Period: ....

Circle the questions or information that would be helpful to solve each Fermi problem.

1. How much food does a school throw out in a month?

   A. There are 2 bones in a chicken wing.

   B. How many garbage bags are filled after each lunch period?

   C. How many bags of recyclables are used after each lunch period?

   D. The school uses 5 buckets of compostable materials for the school garden.

2. How many plastic flamingos are on people's lawns in the United States?

   A. For every 3 people living in apartments, there are 6 people living in a house.

   B. How many people live in the United States of America?

   C. There are 400 million blades of grass per person in the world.

   D. How many flamingos are at the Columbus Zoo and Aquarium?

3. Complete each equation to make it true.

   a $3 \cdot \frac{1}{3} =$ ....

   b $10 \cdot \frac{1}{10} =$ ....

   c $19 \cdot \frac{1}{19} =$ ....

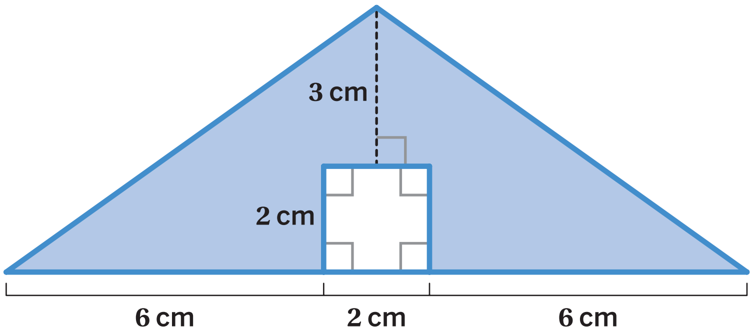
   d $a \cdot \frac{1}{a} =$ .... (as long as a does not equal 0)

   e $5 \cdot$ .... $= 1$

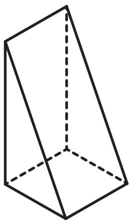
   f $17 \cdot$ .... $= 1$

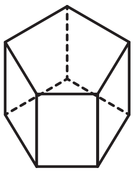
   g $b \cdot$ .... $= 1$

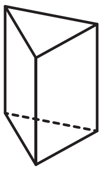
4. Find the area of the shaded region. Show or explain your thinking.

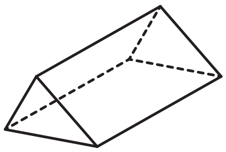


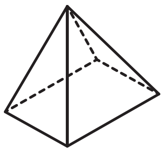
5. Select *all* the figures that are triangular prisms.

A.


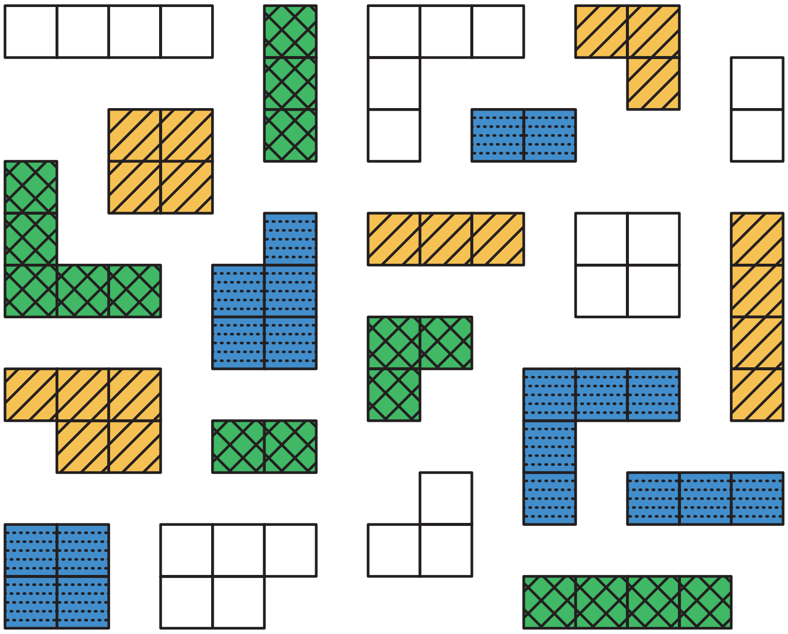
B.


C.


D.


E.


6. Think of different ways you could sort these figures. What categories could you use?



My Notes:

SUB-UNIT

1 What Are Ratios?

Narrative Connections

How does an eggplant become a plum?

Few things are more important to painters than color.

"Color in painting," according to Vincent Van Gogh, "is like enthusiasm in life." In her diaries, Mexican painter Frida Kahlo described herself as "CHROMOPHORE—the one who gives color." Meanwhile French impressionist Claude Monet described color as his "day-long obsession, joy and torment." It is through color that artists give their paintings a sense of life and motion, enabling them—in the words of Georgia O'Keefe—to "say things . . . [they] couldn't say any other way."

In painting, there are three primary colors: red, yellow, and blue. They're called "primary" because they can't be mixed from the *other* colors. They can, however, be combined to create a wide range of other colors. Green, purple, orange, and the shades in between are made from these primary colors. With the right combinations, artists can create light and shadow, or give an image depth. They can draw attention to certain areas of the canvas, or evoke a particular emotion within us.

One of the greatest innovations in color came not from an artist, but from the mathematician Isaac Newton. Newton bent a sunbeam through a prism, creating a rainbow spectrum. He arranged the resulting colors onto a wheel. Over time, this color wheel would evolve into a useful tool to help artists choose meaningful color combinations.

Painters must use these combinations of primary colors in just the right amounts. The wrong combination could mean the difference between blush and cerise, or eggplant and plum! To get the exact right shade, we need a way to express the relationship between the amounts of different pigments.

Unit 2 | Lesson 2

Introducing Ratios and Ratio Language

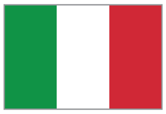
Let's use visuals to describe how quantities relate to each other.

Warm-up Categorizing Flags of the World

Think about how you could group these flags into two or more categories. Be prepared to explain your thinking.


Bosnia and Herzegovina

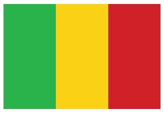

Cambodia


Italy


Kiribati


United Arab Emirates


Bhutan

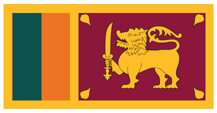

Mali

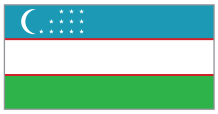

Belgium


Jordan


Papua New Guinea


Uruguay


Sri Lanka


Uzbekistan


Namibia


Romania


Honduras

Public Domain

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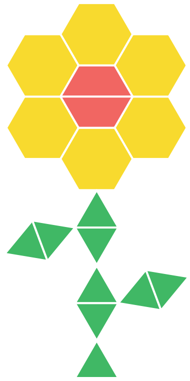
Name: ________________ Date: ________ Period: ________

Activity 1 Ratios and Ratio Language in Flower Patterns

Refer to the flower constructed from pattern blocks.



1. Record the number of each pattern block shape used to make this flower.

Trapezoids: ________________

Hexagons: ________________

Triangles: ________________

2. Complete each statement comparing the number of trapezoids and hexagons.

a There are ________ trapezoids for every ________ hexagons.

b The statement in part a describes the *ratio* of trapezoids to hexagons, which is ________ to ________.

c What is the ratio of hexagons to trapezoids? ________ to ________.

d Write another sentence that describes the ratio of hexagons to trapezoids.

3. Complete each statement comparing the number of triangles and hexagons.

a The ratio of triangles to hexagons is ________ to ________.

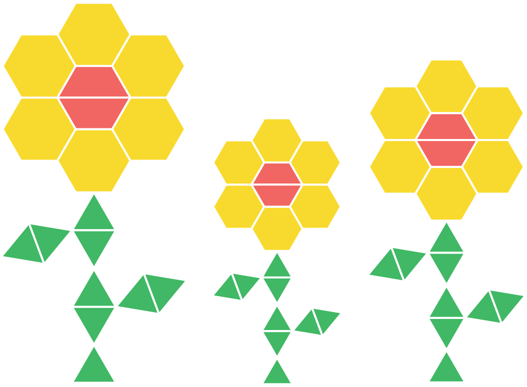
b There are ________ triangles for every ________ hexagons.

c What is the ratio of hexagons to triangles? ________ to ________.

d Write another sentence that describes the ratio of hexagons to triangles.

Activity 2 Ratios and Ratio Language in Flower Gardens

Refer to the flowers constructed from pattern blocks.



1. Record the number of each pattern block shape used to make all three flowers.

   Trapezoids: ....

   Hexagons: ....

   Triangles: ....

2. Write as many sentences as you can think of that describe ratios in this flower garden.
   Hint: There are at least 36 sentences you can write!

Collect and Display: Your teacher will walk around and collect language you use to describe the ratios. This language will be added to a class display for your reference.

3. What do you notice about the ratios that describe a single flower and the ratios that describe the entire flower garden?

Name: ______ Date: ______ Period: ______

Activity 3 Two Truths and a Lie

You will be given a set of instructions to follow for creating a design by using pattern blocks.

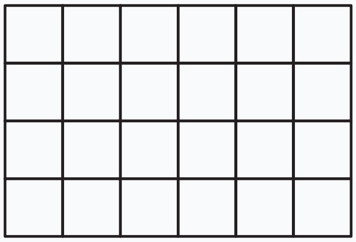
As you *Mix and Mingle* with other groups, record which statement is the lie for each group. Briefly explain why it is a lie.

Group	Lie	Explanation
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Reflect: How did you determine whether a statement was a truth or a lie?

Are you ready for more?

1. Use two colors to shade the rectangle so that there are 2 square units of one color for every 1 square unit of the other color.
2. The rectangle you just shaded has an area of 24 square units. Draw a different shape that does not have an area of 24 square units, but that can also be shaded with two colors in a 2 : 1 ratio. Shade your new shape by using two colors.



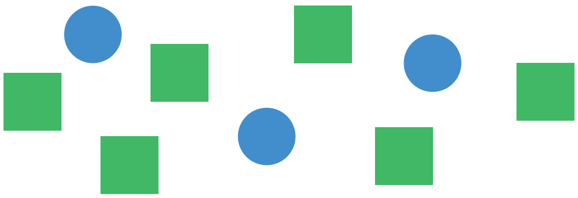
STOP

Summary

In today's lesson . . .

You began to investigate a specific type of relationship between two or more quantities called a ***ratio relationship***.

There are many ways you can describe a situation using *ratio language*. For example, consider this set of squares and circles:



Some statements that describe the relationship between squares and circles using ratio language are:

- The ratio of circles to squares is 3 to 6.
- There are 6 squares for every 3 circles.
- The ratio of circles to squares is $3 : 6$.
- There are 2 times as many squares as there are circles.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. In a fruit basket, there are 9 bananas, 4 apples, and 3 plums.

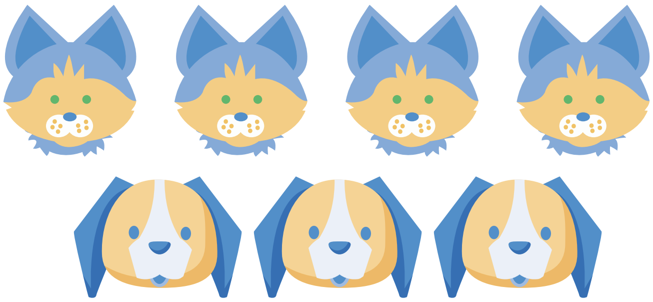
   a. The ratio of bananas to apples is .... : ....

   b. The ratio of plums to apples is .... to ....

   c. For every .... apples, there are .... plums.

   d. For every 3 bananas, there is 1 ....

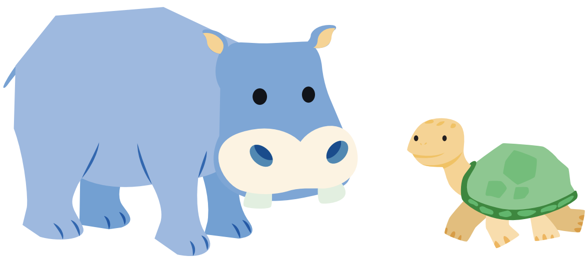
2. Complete the sentences to describe a ratio relationship between the two types of animals in this collection of cats and dogs.



   a. The ratio of dogs to cats is ....

   b. For every .... dogs, there are .... cats.

3. Write two different sentences that use ratios to relate the number of eyes to the number of legs in this picture.



Name: ______ Date: ______ Period: ______

4. Choose an appropriate unit of measurement for each quantity: cm, cm^2, or cm^3.

a. Area of a rectangle

b. Volume of a prism

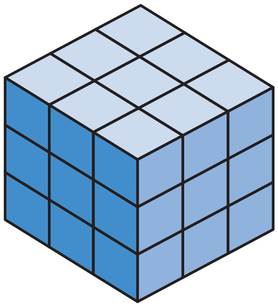
c. Side of a square

d. Area of a square

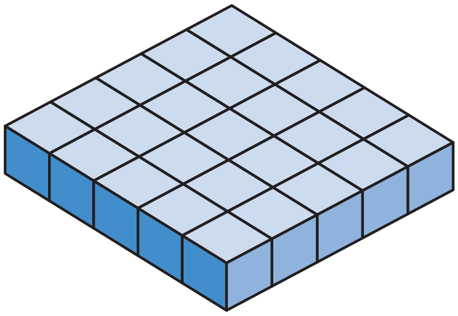
e. Volume of a cube

5. Determine the volume and surface area of each prism.

a. **Prism A:** 3 cm by 3 cm by 3 cm



b. **Prism B:** 5 cm by 5 cm by 1 cm



c. Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?

6. Show at least three different ways you could represent the number 18 using an area model.

Name: .... Date: .... Period: ....

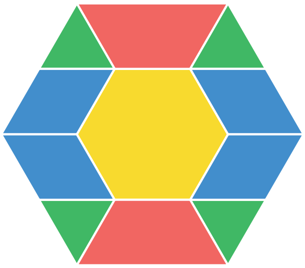
Unit 2 | Lesson 3

Representing Ratios With Diagrams

Let's use diagrams to represent ratios.

Warm-up Pattern Blocks and Ratios

Refer to this design made up of pattern block shapes.



1. Choose two of the shapes in the design, and draw a diagram to represent the ratio of those shapes.

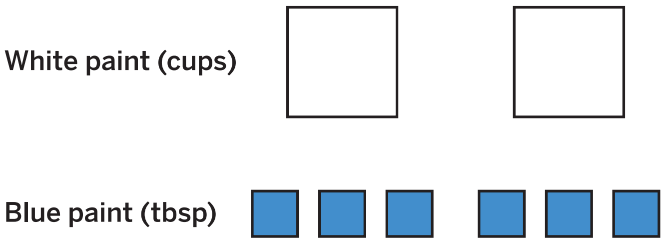
2. Trade books with a partner. On their page, write a sentence to describe a ratio shown in their diagram. Your partner will do the same for your diagram.

3. Return your partner's book. Read the sentence written on your page. If you disagree with the statement, try to rewrite it and explain your thinking to your partner.

Log in to Amplify Math to complete this lesson online.

Activity 1 Mixing Paint

To create a light blue paint, Elena mixed 2 cups of white paint with 6 tablespoons (tbsp) of blue paint.



1. Discuss each statement, and circle *all* those that correctly describe this situation. Make sure that both you and your partner agree with each circled response.

A. The ratio of cups of white paint to tablespoons of blue paint is $2 : 6$.

B. There is 1 cup of white paint for every 3 tbsp of blue paint.

C. There are 3 tbsp of blue paint for every cup of white paint.

D. For every tablespoon of blue paint, there are 3 cups of white paint.

E. For every 6 tbsp of blue paint, there are 2 cups of white paint.

F. For every 3 cups of white paint, there are 7 tbsp of blue paint.

2. Jada also made a light blue paint for an art project by mixing 3 cups of white paint with 9 tablespoons of blue paint.

a Draw a diagram that represents Jada's light blue paint.

b Write at least two sentences describing the ratio of white paint and blue paint that Jada mixed.

Name: .... Date: .... Period: ....

Activity 2 Card Sort: Representing Ratios

You will be given a set of cards describing different amounts of ingredients used in a recipe for guacamole.

1. Take turns with your partner selecting a sentence and matching it with a diagram.

- Explain to your partner how you know the sentence and the diagram match.
- If you disagree with a match your partner presents, explain your thinking and discuss until you reach an agreement.
- Record the number of the sentence that matches each diagram in the table. More than one sentence may match a given diagram.

Diagram	Sentence number
A	
B	
C	
D	
E	
F	

2. After you and your partner have agreed on a match for all of the sentences, compare your matches with the answer key. Discuss any mismatches and update your table with the correct matches.

3. Would guacamole made by using the ratios in Diagrams E and F taste the same? Why or why not?

4. Select one of Diagrams A–D and write another sentence that describes the ratio shown.

Are you ready for more?

If guacamole was made using 4 cloves of garlic, 6 limes, and 12 avocados, would it taste the same as the recipe shown in Diagram F? If not, describe the difference in taste.

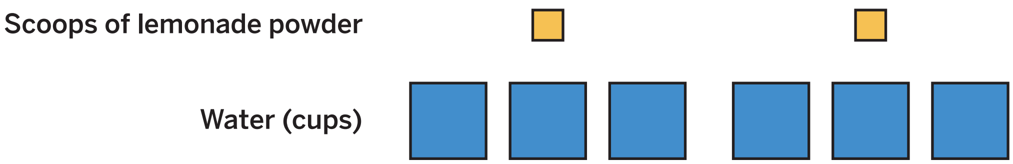
STOP

Summary

In today's lesson . . .

You saw that ratio relationships between quantities can be described using ratio language and can also be represented using diagrams.

For example, a recipe for lemonade, "mix 2 scoops of lemonade powder with 6 cups of water" can be represented using the diagram:



The ratio of scoops of lemonade powder to cups of water is 2 to 6, which can be written as $2 : 6$.

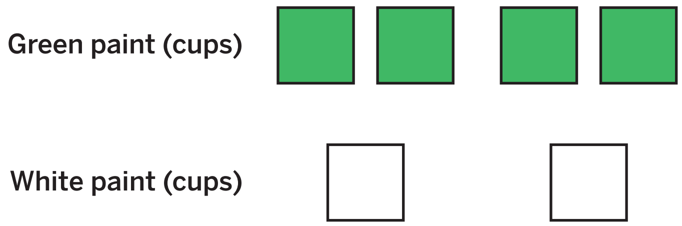
You used diagrams to reason about other ways the relationship between two quantities can be described. For example, you could also say that every scoop of lemonade powder corresponds to 3 cups of water, which can be written as the ratio $1 : 3$.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. The diagram represents the cups of green paint and cups of white paint in a paint mixture. Select *all* the statements that correctly describe the relationship between green paint and white paint.



A. The ratio of cups of white paint to cups of green paint is 2 to 4.

B. For every cup of green paint, there are 2 cups of white paint.

C. The ratio of cups of green paint to cups of white paint is 4 : 2.

D. For every cup of white paint, there are 2 cups of green paint.

E. The ratio of cups of green paint to cups of white paint is 2 : 4.

2. A recipe for snack mix says to combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

a. Create a diagram to represent the amounts of each ingredient in this recipe.

b. Use your diagram to complete each sentence.

The ratio of pretzels to almonds is .... : ....

There are .... cups of pretzels for every 1 cup of raisins.

There are .... cups of almonds for every 1 cup of raisins.

Practice

Name: ______ Date: ______ Period: ______

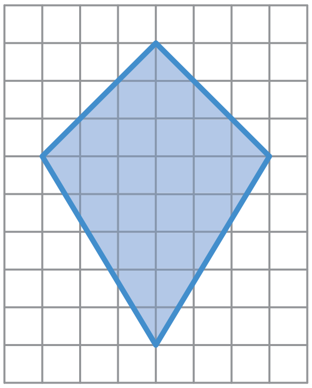
3. Determine the stated measurements for the squares described. Include the appropriate units.

a A square is 3 in. by 3 in. What is its area?

b A square has a side length of 5 ft. What is its area?

c The area of a square is 36 cm^2. What is the length of each side of the square?

4. Determine the area of this quadrilateral. Show or explain your strategy.



5. Evaluate each expression.

a $\frac{1}{8} \cdot 8$

b $\frac{1}{8} \cdot 7$

c $\frac{3}{8} \cdot 8$

d $\frac{3}{8} \cdot 7$

6. Mai is creating goodie bags for her birthday party. The table shows the number of items in one goodie bag.

Bracelets	Animal erasers	Stickers
1	3	12

a Determine the number of animal erasers Mai would need to make 8 goodie bags.

b Determine the number of bags Mai made if she used 72 stickers.

Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 4

A Recipe for Purple Oobleck

Let's explore ratios in recipes.

Warm-up Number Talk

Mentally evaluate each expression. Be prepared to explain your thinking.

1. $6 \cdot 15$

2. $12 \cdot 15$

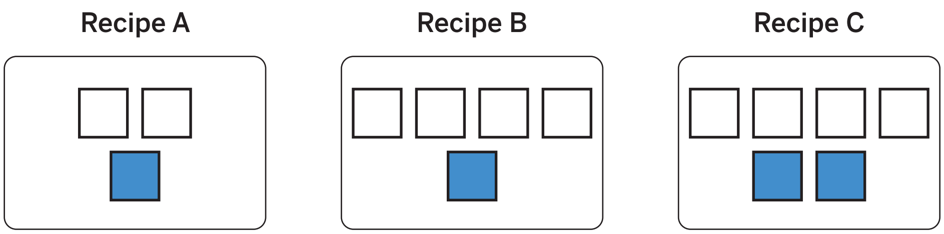
3. $6 \cdot 45$

4. $13 \cdot 45$

Log in to Amplify Math to complete this lesson online.

Activity 1 Making Oobleck

Oobleck **is a substance called a suspension, which can mimic the qualities of both a solid and a liquid. Here are diagrams representing three possible recipes for making oobleck using cornstarch and water.**



Key:

□ **= 1 cup of cornstarch** ■ **= 1 cup of water**

1. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe B?

2. Use the diagrams to complete each pair of statements.

a Recipe A uses ________ cup(s) of cornstarch and ________ cup(s) of water. The ratio of cups of cornstarch to cups of water in Recipe A is ________.

b Recipe C uses ________ cup(s) of cornstarch and ________ cup(s) of water. The ratio of cups of cornstarch to cups of water in Recipe C is ________.

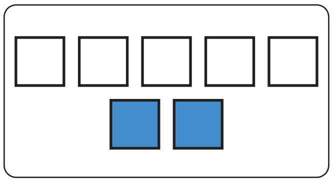
3. How might the texture of oobleck made from Recipe A compare to the texture of oobleck made from Recipe C?

Name: Date: Period:

Activity 1 Making Oobleck (continued)

4. Refer to Recipe D shown here.

Recipe D



Key:

= 1 cup of cornstarch

= 1 cup of water

a Write the ratio of cornstarch to water in Recipe D.

b Describe the consistency of oobleck made from Recipe D.

c What could be done to “fix” Recipe D so that the oobleck made will have the same consistency as oobleck made from Recipe A? **Note:** You cannot remove any ingredient that is already added to the mixture. You can only add ingredients.

d Using your fix, write a ratio for a new Recipe E so that oobleck made from Recipe E has the same consistency as oobleck made from Recipe A.

5. What do you notice about the ratios for Recipes A, C, and E?

Activity 2 Coloring Your Oobleck

When mixing colors, ratios can tell you when two results should be the same. However, not everyone sees colors the same way. There are several reasons for this – one reason is that most people (called *trichromats*) have three types of retinal cone cells, while some (called *tetrachromats*) have four types. Trichromats can see around 1 million different colors, while tetrachromats can see as many as 100 million colors!

Researchers like Dr. Kimberly A. Jameson study how people experience colors differently by presenting different ratios of colors mixed together for subjects to identify and categorize.

Now imagine you are running color-matching experiments of your own, using dyed oobleck. To color one batch of oobleck purple, you can add 2 red drops and 5 blue drops of food coloring to water.

1. What is the ratio of red drops to blue drops of food coloring for one batch?

2. Draw a diagram showing the number of red drops related to the number of blue drops that would make *double* the amount of food coloring. Then write these amounts as a ratio.

3. How do you know that this will make the exact same purple?

Featured Mathematician

Kimberly A. Jameson

Kimberly A. Jameson is a Project Scientist at UC Irvine's Institute for Mathematical Behavioral Sciences. She has conducted numerous research studies on the perception of color, human tetrachromacy, and why individuals "see" colors differently.

Kimberly A. Jameson

Name: .... Date: .... Period: ....

Activity 2 Coloring Your Oobleck (continued)

4. Write the ratio of the number of red drops to the number of blue drops of food coloring that are needed to *triple* the mixture of food coloring. Explain your thinking.

5. How many batches of oobleck can you color with 10 drops of red food coloring and 25 drops of blue food coloring?

6. Find another ratio of red drops to blue drops that would produce the same purple color.

7. How many batches of oobleck can you color using this new ratio of red to blue drops?

Are you ready for more?

Sports drinks use sodium (better known as salt) to help people replenish electrolytes. Here are the nutrition labels of two sports drinks.

Sports drink A

Nutrition Facts
Serving Size 8 fl oz (240 mL)
Serving Per Container 4

Amount Per Serving
Calories 50

		% Daily Value*
Total Fat	0 g	0%
Sodium	110 mg	5%
Potassium	30 mg	1%
Total Carbohydrate	14 g	5%
Sugars	14 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

Sports drink B

Nutrition Facts
Serving Size 12 fl oz (355 mL)
Serving Per Container about 2.5

Amount Per Serving
Calories 80

		% Daily Value*
Total Fat	0 g	0%
Sodium	150 mg	6%
Potassium	35 mg	1%
Total Carbohydrate	21 g	7%
Sugars	20 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

1. Which of these drinks is saltier? Explain your thinking.

2. If you wanted to make sure a sports drink was less salty than both of these drinks shown here, what ratio of sodium to water would you use?

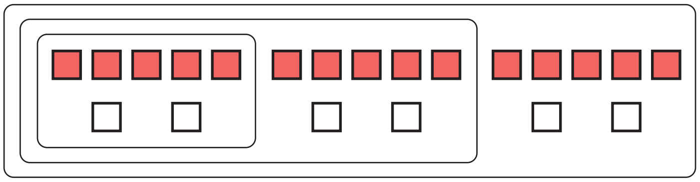
STOP

Summary

In today's lesson . . .

You explored different combinations of cornstarch and water, and red or blue food coloring. You were able to create different textures and different colors. You realized that some combinations created the same texture or color and compared the *ratio* of their ingredients using diagrams and numeric values. A ***ratio*** is a comparison of two quantities, such that for every a units of one quantity, there are b units of another quantity.

The diagram shows the *ratio* of red paint to white paint in a single batch, double batch, and triple batch of a recipe.



Single batch: $5 : 2$.

Double batch: $10 : 4$

Triple batch: $15 : 6$.

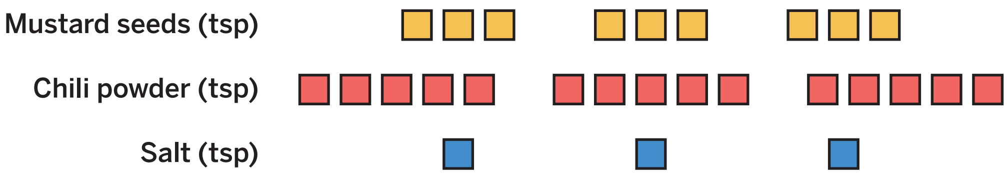
These ratios are ***equivalent*** because they all represent the same pink color (or the same ratio of red paint to white paint).

Reflect:

Name: .... Date: .... Period: ....

1. Priya makes chocolate milk by mixing 2 cups of milk and 5 tbsp of cocoa powder. Draw a diagram that clearly represents doubling her recipe for chocolate milk.

2. A recipe for 1 batch of spice mix says, "Combine 3 tsp of ground mustard seeds, 5 tsp of chili powder, and 1 tsp of salt." How many batches are represented by the diagram? Show or explain your thinking.



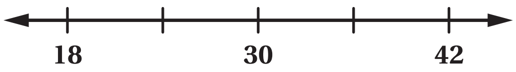
3. In a recipe for sparkling grape juice, the ratio of cups of sparkling water to cups of grape juice concentrate is 3 to 1.

a. Find two more ratios of cups of sparkling water to cups of juice concentrate that would make larger amounts of sparkling grape juice that each tastes the same as this recipe.

b. Write a ratio of sparkling water and grape juice that would *not* taste the same as this recipe. Then describe how it would taste different.

Name: ______ Date: ______ Period: ______

4. The tick marks on this number line are equally spaced. Write the missing numbers under the two unlabeled tick marks on the number line.



5. At the kennel, there are 6 dogs for every 5 cats.

a The ratio of dogs to cats is ______ to ______.

b For every ______ cats there are ______ dogs.

c Would the ratio $6 : 5$ represent the relationship between the total number of dogs and the total number of cats? Explain your thinking.

d Would the ratio $6 : 5$ represent the relationship between the total number of cats and the total number of dogs? Explain your thinking.

6. What is $\frac{1}{4}$ of 100? Show or explain your thinking.

Name: Date: Period:

Unit 2 | Lesson 5

Kapa Dyes

Let's see how mixing colors relates to ratios.

Warm-up Number Talk

Mentally evaluate each expression.

1. $24 \div 4$

2. $\frac{1}{4} \cdot 24$

3. $24 \div \frac{4}{1}$

4. $5 \div 4$

Log in to Amplify Math to complete this lesson online.

Activity 1 'Uki'uki/Ma'o Dye

Artist Dalani Tanahy specializes in *kapa*, the traditional native Hawaiian technique for making cloth using tree bark. An important part of the process is decorating kapa with designs and patterns using natural dyes. Specific ratios of two dyes are mixed to create new colors. For example, greens from the maʻo plant and blues from the 'uki'uki berry combine to make a teal dye.

Combining blue and green food coloring or paint can replicate the specific teal color — the 'uki'uki/ma'o dye — shown on the wheel.

Photo courtesy of Dalani Tanahy

1. A mixture for 'uki'uki/ma'o dye uses a ratio of green to blue of $15 : 45$. Draw a diagram to represent this mixture.

Plan ahead: In what ways will you show respect for other cultures as you complete this activity?

2. Think about making a smaller amount of the same color of the 'uki'uki/ma'o dye.

a What is one way you could make a smaller amount that is the same color? Show or explain your thinking.

b Is the ratio in your mixture from Problem 2a the same as the ratio in the original mixture? Explain your thinking using ratios and ratio language.

Name: ________ Date: ________ Period: ________

Activity 1 'Uki'uki/Ma'o Dye (continued)

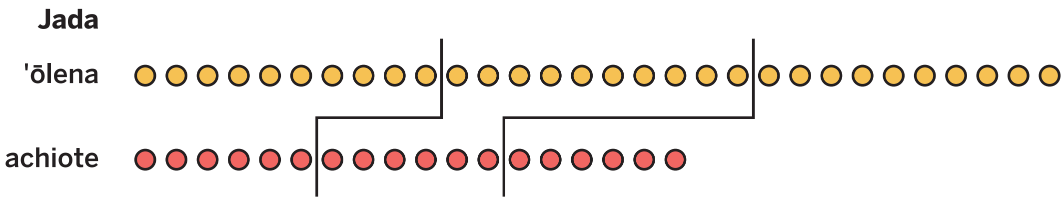
3. Write the ratios from Problems 1 and 2 in the table. Describe any patterns you notice.

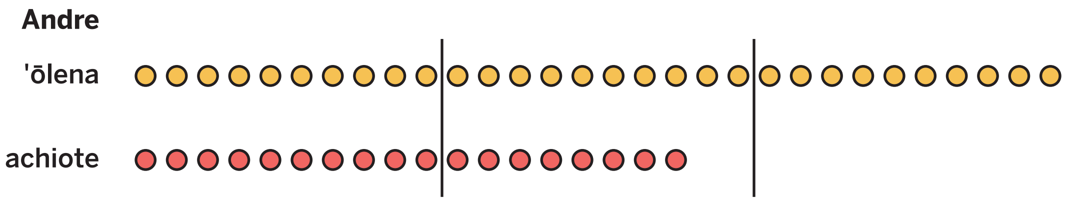
Ma'o green	'Uki 'uki blue

Activity 2 'Ōlena/Achiote Dye

The orange dye seen on the color wheel is traditionally made by combining yellow from the 'ōlena (turmeric) plant and red from the seeds of the achiote plant. A mixture for 'ōlena/achiote dye calls for 30 ml of 'ōlena yellow with 18 ml of achiote red.

1. Jada and Andre each attempted to make a smaller amount of the same 'ōlena/achiote color using food coloring. Jada mixed 10 ml of 'ōlena yellow with 6 ml of achiote red. Andre mixed 5 ml of 'olena yellow with 5 ml of achiote red. Diagrams that represent their color mixtures are shown.





a Does either person's color mixture make the same color orange as the known 'ōlena/achiote mixture? Explain your thinking.

b If either person's mixture did not produce the same color orange, what might they have done incorrectly?

Name: .... Date: .... Period: ....

Activity 2 'Ōlena/Achiote Dye (continued)

2. Describe one other way you could combine different amounts of 'ōlena yellow and achiote red that would result in the same orange color as the original mixture but produce a *smaller amount*. Show or explain your thinking.

3. Complete the table with the possible ratios for making the known 'ōlena/achiote dye.

'Ōlena yellow	Achiote red
30	18

STOP

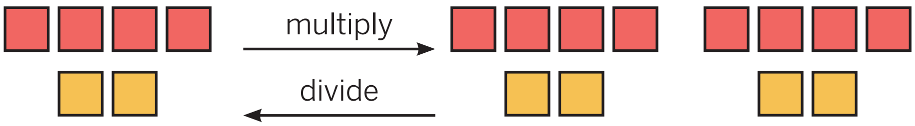
Summary

In today's lesson . . .

You saw again that when mixing colors, you can use *ratios* to determine different amounts of each color that can be combined to create the same color.

To make *larger* amounts, you can always *multiply* the amount of each color by the same number (greater than 1) and the color will be the same.

To make smaller amounts, you can always *divide* the amount of each color by the same number (or multiply by the same fraction), and the color will be the same.



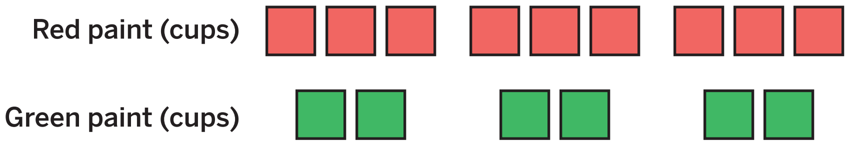
$4 : 2$ $8 : 4$

Both groups represent a ratio of $4 : 2$ and makes the same color orange paint. Ratios $4 : 2$ and $8 : 4$ are *equivalent* because in each ratio the first value is double the second value.

Reflect:

Name: .... Date: .... Period: ....

1. The diagram shows a mixture of red paint and green paint needed for 3 batches of a particular brown paint. How could you show 1 batch of the same brown paint? What is the ratio of red paint to green paint, for 1 batch?



2. Diego makes green paint by mixing 10 tbsp of yellow paint and 2 tbsp of blue paint. Which of these mixtures produce the same green paint as Diego's mixture, but in a smaller amount? Select *all* that apply.

A. For every 5 tbsp of blue paint, mix 1 tbsp of yellow paint.

B. Mix tablespoons of yellow paint and blue paint in the ratio $5 : 1$.

C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.

D. Mix 11 tbsp of yellow paint and 3 tbsp of blue paint.

E. For every tablespoon of blue paint, mix 5 tbsp of yellow paint.

3. To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.

a Clare only needs half the amount of sky blue paint. What ratio would represent half the recipe?

b Explain how to make a mixture that is a darker tint of blue than the sky blue.

c Explain how to make a mixture that is a lighter tint of blue than the sky blue.

Practice

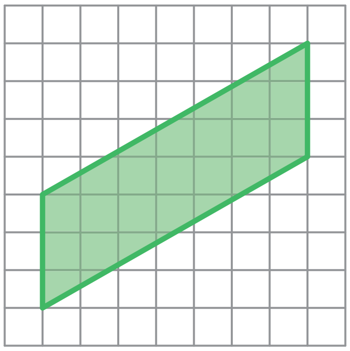
Name: .... Date: .... Period: ....

4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas and 1 tbsp of cocoa powder.

   a. Create a diagram to represent the quantities of each ingredient in the recipe.

   b. Write *three* different sentences that use ratio language to describe the recipe.

5. Determine the area of the parallelogram. Show your thinking.



6. Evaluate each product.

   a. $3 \cdot \frac{2}{3}$

   b. $\frac{7}{5} \cdot 5$

   c. $2 \cdot \frac{3}{4}$

SUB-UNIT

2 Equivalent Ratios

Narrative Connections

How do you put your music where your mouth is?

Antoinette Clinton was just 20 years old when she took the stage in Leipzig, Germany. Better known by her stage name, Butterscotch, she was born in Sacramento, California to a musical family. Her mother was a piano teacher. Her siblings played trumpet, cello, clarinet, and trombone. But tonight was the night of the first Beatbox Battle World Championship. She had come to showcase a different musical instrument: herself!

Beatboxing has long been a core element of hip-hop. Pioneered by artists like Doug E. Fresh, Biz Markie, and Darrell "Buffy" Robinson, performers use their mouth, throat, and nose to imitate a drum kit. MC's would then rap over their beats.

More than 20 years later, beatboxing re-emerged as an international phenomenon. In 2005, Butterscotch was crowned the first Individual Female Beatbox Battle World Champion. Two years later, she beat out 18 men to become the West Coast beatboxing champion.

To be a champion beatboxer, you need a strong sense of timing. An artist needs to know the length of each of their "hits", as well as how many "hits" they can fit into a measure of music. Ratios give performers a way to conceptualize and map those hits so that they never miss a beat.

Unit 2 | Lesson 6

Defining Equivalent Ratios

Let's investigate equivalent ratios.

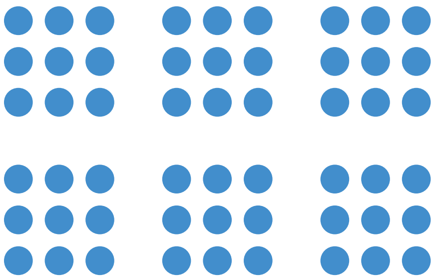
Warm-up Dots and Half Dots

Determine the number of dots in each image. Be prepared to explain your thinking.

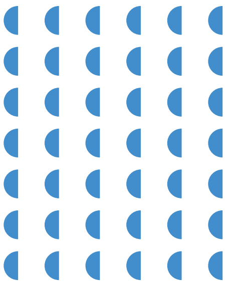
Key:

● $= 1$

Dot Pattern 1



Dot Pattern 2



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Name: ________ Date: ________ Period: ________

Activity 1 Clapping a Rhythm

Notes are used in music to make systematic arrangements of sound, called *rhythms*. Different notes indicate how long a sound is played — the number of *counts* for which a note is held. Some notes are shorter and some notes are longer. Several shorter notes create a faster sounding rhythm, while longer notes create a slower sounding rhythm.

Here are the notations for representing three types of notes in a musical composition.

Eighth Note


$= \frac{1}{2}$ count

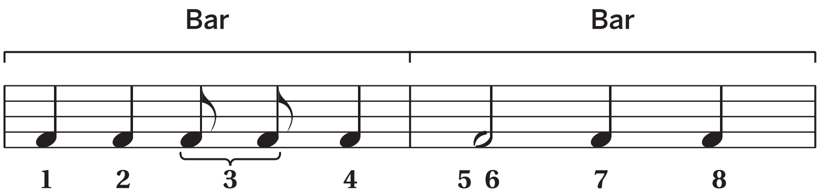
Quarter Note


$= 1$ count

Half Note


$= 2$ counts

The composition of notes shown here has two sections, called *bars*. Your group will be assigned a count 1, 2, 3 or 4. When directed, follow the counts and clap your part according to the notes assigned to your count in each bar.



1. How many notes are in each bar?

2. How many counts are in each bar?

3. How many counts would you expect to be in a third bar?

4. Complete three more rows of this table showing the number of counts for different numbers of bars.

5. Do the counts and bars represent a ratio relationship? Explain or show your thinking.

Compare and Connect: Compare with your group how you generated the values in your table, paying close attention to your reasoning.

Counts	Bars
4	1

Activity 2 What Are Equivalent Ratios?

The ratios $5 : 3$ and $10 : 6$ are *equivalent ratios* because they describe the same ratio relationship. Complete the ratio box to show this is true.

5	3

1. Determine whether each ratio is also equivalent to $5 : 3$ and $10 : 6$. Show or explain your thinking using a ratio box.

a $15 : 12$

5	3

b $30 : 18$

5	3

2. Determine two additional ratios that are equivalent to $5 : 3$. Show your thinking by using ratio boxes.

5	3

5	3

3. Write a definition for *equivalent ratios*.

4. How do you know when two ratios are *not* equivalent?

STOP

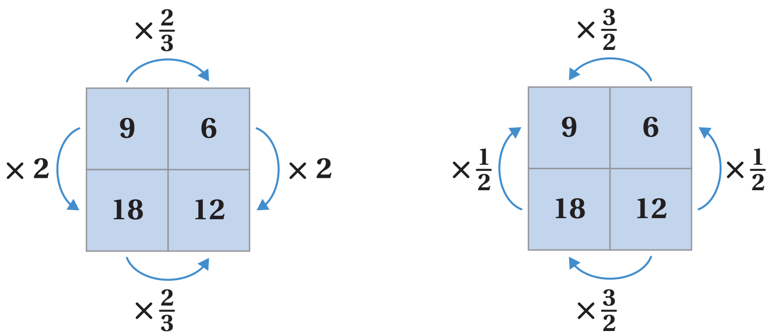
Name: ______________________ Date: ____________ Period: __________

Summary

In today's lesson . . .

You saw that ***equivalent ratios*** are two ratios where the values for one quantity in each ratio can be multiplied or divided by the same number to generate the values for the second quantity in each ratio. For example, the ratios $9:6$ and $18:12$ are equivalent because $9 \cdot \frac{2}{3} = 6$ and $18 \cdot \frac{2}{3} = 12$.

You used a *ratio box* to show and generate *equivalent ratios*.



Reflect:

Practice

Name: ____________ Date: ____________ Period: ____________

1. Here are four pairs of equivalent ratios. Explain or show, such as by drawing a ratio box, how you know each pair of ratios is equivalent.

   a. $4 : 5$ and $8 : 10$

   b. $18 : 3$ and $6 : 1$

   c. $2 : 7$ and $10{,}000 : 35{,}000$

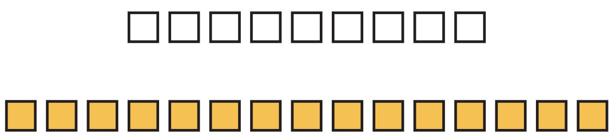
   d. $24 : 18$ and $8 : 6$

2. Explain why $6 : 4$ and $18 : 8$ are *not* equivalent ratios.

3. Do the ratios $3 : 6$ and $6 : 3$ describe the same relationship? Why or why not?

Name: ______ Date: ______ Period: ______

4. This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.



5. In a fruit bowl there are 6 bananas, 4 apples, and 3 oranges. Complete the statements about the ratios of types of fruit in the bowl.

a For every 4 ______, there are 3 ______.

b The ratio of ______ to ______ is $6 : 3$.

c The ratio of ______ to ______ is $4 : 6$.

d For every 1 orange, there are ______ bananas.

6. The table shows the number of cups of flour needed to make two batches of a recipe for banana bread. Complete the table by adding three more rows of ratios that are equivalent to $4 : 2$.

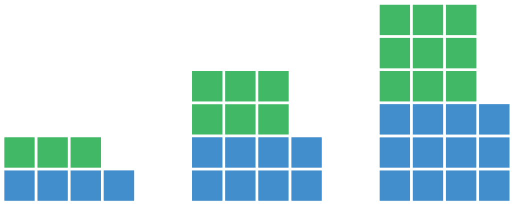
Flour	Batches
4	2

Representing Equivalent Ratios With Tables

Let's use tables to represent equivalent ratios.

Warm-up How Is It Growing?

Look for a pattern in the set of figures shown.



1. How many total squares will be in:

a. Figure 4?

b. Figure 5?

c. Figure 10?

2. Describe how you see the pattern growing.

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 Jazz Rhythm and Horn Sections


Jim Katz

In April 2020, the jazz world lost an icon when Ellis Marsalis, Jr. passed away from COVID-19. Ellis was the patriarch of the legendary Marsalis family, and he left behind a tremendous legacy, from his recordings, to the Ellis Marsalis Center for Music in his beloved hometown of New Orleans, to his teachings and many former students. Perhaps the most noteworthy students being several of his sons, accomplished jazz musicians in their own rights – with multiple Grammy Awards. Ellis (piano) and four of his six sons, Branford (saxophone), Wynton (trumpet), Delfeayo (trombone), and Jason (drums) are considered the "first family of jazz" for good reason.

A jazz orchestra, also called a big band, typically consists of a horn section made up of 5 saxophones, 4 trumpets, 4 trombones, and a rhythm section featuring a piano, a guitar, a bass, and drums.

1. What is the ratio of the number of members of the horn section to the number of members of the rhythm section in a typical jazz orchestra?

2. Use a ratio box to show how you know whether the ratio of the number of horn players to the number of rhythm players in the Marsalis family is equivalent to that in a typical jazz orchestra.

3. Imagine several jazz orchestras get together for a concert. Complete this ratio table with different possible equivalent ratios of the number or horn players to the number of rhythm players for three different-sized orchestras.

Horn players	Rhythm players
13	4

Activity 2 Beignet Recipe

A large family tripled a beignet recipe and used 3 cups of evaporated milk, 21 cups of flour, a half dozen eggs, $4\frac{1}{2}$ cups of warm water, $1\frac{1}{2}$ cups of sugar, $\frac{3}{4}$ of a cup of shortening, 6 tsp of active yeast, and 3 tsp of salt.

1. Determine four equivalent ratios for the amounts of flour and milk needed to make different-sized batches of the same beignet recipe: two that use *more* flour and milk, and two that use *less* flour and milk.

2. What method(s) did you use to determine the equivalent ratios using more ingredients? Less ingredients?

3. How do you know that each row shows a ratio that is equivalent to your original ratio? Show or explain your thinking.

Are you ready for more?

You have created a best-selling recipe for lemon scones. The ratio of sugar to flour is $2:5$. Use a separate sheet of paper to create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amount using more than 500 cups of flour.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You saw that you can add column headers to a *ratio box* and extend it down by adding more rows to represent multiple sets of *equivalent ratios*. This is then called a ***ratio table***.

You can use a ratio table in the same ways you used a ratio box — to determine or verify *equivalent ratios*.

This table shows the price of different number of mangos.

The values in each row can be determined by multiplying the corresponding values in each previous row by some same number.

Price ($)	Number of mangos
2	3
4	6
6	9
8	12
10	15

$\times 2$, $\times \frac{3}{2}$, $\times \frac{4}{3}$, $\times \frac{5}{4}$

$\times \frac{3}{2}$

Notice that each row in the table shows that the ratio of number of mangos to the total cost is $3:2$, which means that each value in the number of mangos column is 1.5 (or $\frac{3}{2}$) times the corresponding cost in dollars from the same row.

Reflect:

Name: .... Date: .... Period: ....

1. A particular orange paint is made by mixing 5 parts of yellow paint with 6 parts of red paint.

a. Complete the table with the amounts of yellow paint and red paint needed to make different amounts of the same shade of orange paint.

Yellow paint (parts)	Red paint (parts)
5	6

b. Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange paint.

2. A car travels at a constant speed and its distance traveled in 1, 2, and 3 hours is shown in the table. How far does the car travel in 12 hours? Explain or show your thinking.

Time (hours)	Distance (km)
1	70
2	140
3	210

Name: .... Date: .... Period: ....

Practice

3. In a recipe for scones, there is 1 cup of milk for every 3 cups of flour. A baker needs to make 5 batches of scones. Determine how much of each ingredient the baker will need. Consider using a table to help with your thinking.

a How many cups of milk are needed to make 5 batches of scones?

b How many cups of flour are needed to make 5 batches of scones?

4. The tick marks on the number line are equally-spaced apart. Write fractions to represent the values of locations A and B on the number line.

5. Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water. Determine *two* more equivalent ratios of liters of orange juice to liters of soda water that would make sparkling orange juice that tastes the same as Noah's recipe.

6. List *all* the factors of 20.

Unit 2 | Lesson 8

Reasoning With Multiplication and Division

Let's use multiplication and division to go from a given number to any other number.

Warm-up Problem Strings

Mentally determine each product and quotient.

Expression	Product
$7 \cdot 3$	
$7 \cdot 6$	
$7 \cdot 30$	
$7 \cdot 15$	
$7 \cdot 18$	
$14 \cdot 12$	
$14 \cdot 48 \cdot 25$	

Expression	Quotient
$210 \div 21$	
$210 \div 42$	
$1{,}260 \div 84$	
$1{,}260 \div 7$	
$630 \div 35$	
$210 \div 35$	
$210 \div 7 \div 5 \div 3$	

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Divide and Multiply, Multiply and Divide

Write a sequence of operations that connects each starting number to the corresponding target number by using only multiplication and division. An example is provided in the first row of the table.

Starting number	Sequence of operations	Target number
6	$\div 3 \cdot 2$	4
12		20
60		50
24		9
5		8

Are you ready for more?

Write a sequence of operations that connects the starting number $\frac{2}{3}$ to the target number $\frac{1}{4}$ by using only multiplication and division.

Activity 2 Two Operations, One Operation

For Problems 1 and 2, refer to the table that shows several starting numbers and their corresponding target numbers.

1. Write a sequence of *exactly two operations* that connects each starting number to the corresponding target number in the table by using only multiplication and division.

2. Write *one operation*, by using either multiplication or division, that connects each starting number to the corresponding target number in the table.

Starting number	Two operations	One operation	Target number
5			4
12			17
123			987
848			484

Are you ready for more?

Think about any two numbers, calling the starting number a and the target number b.

1. Write an expression representing a sequence of two multiplication and division operations that connects a to b.

2. Write one multiplication or division operation that also connects a to b.

STOP

Name: Date: Period:

Summary

In today's lesson . . .

You revisited important relationships between multiplication and division and how they relate to fractions. You reasoned that when completing one division step and one multiplication step they can be completed in either order, or even be rewritten using one operation.

You applied this understanding to reason about how to apply division and multiplication to connect two values. You determined that to get from the first value to the second value you can divide by the first value then multiply by the second value. You can simplify this into a single multiplication expression using the relationship between division and fractions.

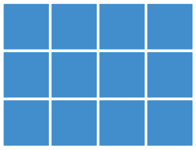
Starting value	Two operations	One operation	Target value
9	$\div 9 \cdot 23$	$\cdot \frac{9}{23}$	23

Reflect:

Practice

Name: .... Date: .... Period: ....

1. This area model can be used to represent the product $3 \cdot 4$ and also the quotient $12 \div 4$.



a. How could the model be adjusted to represent the product $6 \cdot 4$?

b. How could the model be adjusted to represent the quotient $6 \div 4$?

2. Write an expression to represent each statement.

a. Multiply 11 by the quotient of 5 and 6.

b. Divide 20 by the product of 3 and 7.

3. Determine whether each expression is greater than, less than, or equal to $\frac{4}{7}$. Explain or show your thinking.

a. $\frac{9}{10} \cdot \frac{4}{7}$

b. $\frac{4}{7} \cdot \frac{10}{9}$

c. $\frac{8}{14}$

Name: .... Date: .... Period: ....

Practice

4. The surface area of a cube is equal to $\frac{90}{41}$ cm^2. What is the area of one face of the cube? Explain or show your thinking.

5. Complete the table to determine at least two equivalent ratios to $42 : 30$ with lesser values and at least two equivalent ratios with greater values.

42	30

6. Label each number as *prime* or *composite*. If the number is *composite*, list as many factors as you can. If the number is *prime*, explain your thinking.

a 24

b 31

c 93

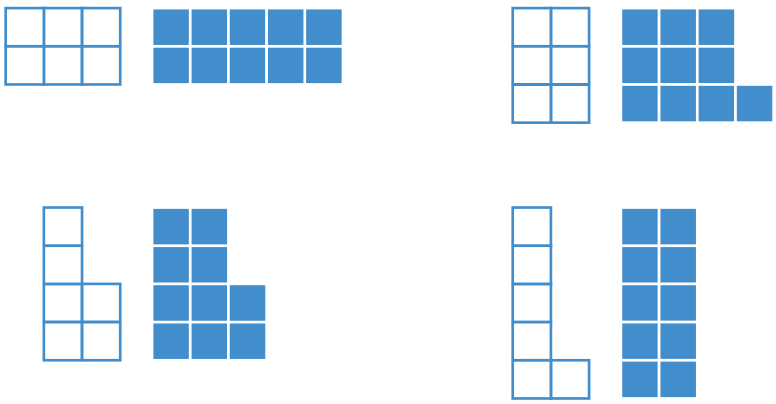
d 2

Common Factors

Let's use factors to solve problems.

Warm-up Figures Made of Squares

Study these four pairs of figures. How are the pairs of figures similar? How are they different?



Similarities	Differences

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Percussion Camp

The percussion section of a university marching band consists of 12 snares, 10 cymbals, 8 bass drums, 4 timpani drums, and 4 tenor drums (also called quads). During summer practice, smaller groups will break off to rehearse.

1. How could the snares and bass drums be grouped so there is the same number of each instrument in every group?

2. How could the cymbals and timpani drums be grouped so that there is the same number of each instrument in every group?

3. Could the entire percussion section be placed into smaller groups so that each group includes the same number of each instrument? If so, how?

Are you ready for more?

Several percussion sections are getting together to practice a song for a parade. There are 24 gong players and 16 triangle players.

What is the greatest number of smaller groups that they could be arranged into where each group has the same number of gong players and the same number of triangle players?

Activity 2 Greatest Common Factor

Not all musicians think about their music as being related to math, even though it likely is. Jazz drummer Clayton Cameron on the other hand once heard his music referred to as "some beautiful numbers" (as in musical numbers) and ever since he has not stopped thinking about that relationship. He has even coined the term *a-rhythm-etic* to describe the "cycles and groupings of numbers and how they feel." Musical cycles are closely related to greatest common factor.

1. The "greatest common factor" of 30 and 18 is 6. What do you think the term *greatest common factor* means?

2. Determine *all* of the common factors of 21 and 6. Then identify the greatest common factor.

3. Determine the greatest common factor of each pair of numbers.

a 28 and 12

b 35 and 96

Featured Mathematician

Clayton Cameron

Clayton Cameron is a native of Los Angeles and is a lecturer on Global Jazz Studies at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, Cameron became a rising star in the music industry, performing as a percussionist with countless award-winning acts. He is particularly known for perfecting "the art of the brush technique," which he did by treating it more as a science of numbers.

Name: .... Date: .... Period: ....

Activity 2 Greatest Common Factor (continued)

4. A small rectangular bulletin board is 12 in. tall and 27 in. wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

a What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your thinking.

b How is the solution to this problem related to the greatest common factor?

Are you ready for more?

A school has 1,000 lockers, all lined up in a hallway. Each locker is closed. Then . . .

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open?

STOP

Summary

In today's lesson . . .

You reviewed that a *factor* of a whole number is another whole number that divides into the given number evenly (with no remainder). Given any two whole numbers, you reasoned that you could determine their ***common factors*** and their ***greatest common factor*** (GCF).

Two whole numbers can have one or many common factors, but will only ever have one GCF.

Numbers	Factors	Greatest common factor
45 60	1 3 5 9 15 45 1 2 3 4 5 6 10 12 15 20 30 60	15
20 81	1 2 4 5 10 20 1 3 9 27 81	1

Reflect:

Name: ______ Date: ______ Period: ______

Practice

1. In your own words, what does the term *greatest common factor* mean? Describe a process for determining the greatest common factor of two numbers.

2. A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers each. What are some other possibilities? Explain or show your thinking.

3. A school chorus has 90 sixth-grade students and 75 seventh-grade students. The music director wants to make groups of performers, with the same combination of sixth- and seventh-grade students in each group. She wants to form as many groups as possible.

a What is the greatest number of groups that could be formed? Explain or show your thinking.

b Using your answer from Problem 3a, how many students of each grade would be in each group?

Practice

Name: ______ Date: ______ Period: ______

4. Complete each statement about a class that has 4 boys for every 3 girls.

a) The ratio of boys to girls is ______ to ______.

b) The ratio of girls to boys is ______ to ______.

c) For every ______ boys there are ______ girls.

d) The ratio of girls to boys is ______ : ______.

5. Clare makes purple paint by mixing 6 tbsp of blue paint and 4 tbsp of red paint. Which of these mixtures produces the same purple paint as Clare's mixture? Select all that apply.

A. Mix tablespoons of blue paint and red paint in the ratio of $3 : 2$.

B. For every 3 tbsp of red paint, mix 2 tbsp of blue paint.

C. Mix tablespoons of blue paint and red paint in the ratio of $9 : 6$.

D. For every 2 tbsp of red paint, mix 3 tbsp of blue paint.

E. Mix 7 tbsp of blue paint and 5 tbsp of red paint.

6. List all of the multiples of 5 less than or equal to 50.

Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 10

Common Multiples

Let's use multiples to solve problems.

Warm-up Keeping a Steady Beat

Part 1

You will be given instructions for making a rhythm as a class. As you play your part, think about this question.

1. When will the two sounds happen at the same time?

Part 2

You will be given new instructions for making a different rhythm. As you play your part, think about these questions.

2. When will the two sounds happen at the same time?

3. What would happen if you kept on playing the rhythm?

Log in to Amplify Math to complete this lesson online.

Activity 1 A New Rhythm

Plan ahead: What choices will you make to control your impulses as you participate in making a rhythm?

Part 1

You will be given instructions for another new rhythm. As you play your part, think about these questions.

1. When will the three sounds happen at the same time (up to 36)?

2. When is the first time that the two sounds happen at the same time?

Part 2

Let's explore common multiples some more.

Suppose in a new rhythm, you clap every 4 beats, stomp every 8 beats, and say "yeah" every 12 beats.

3. When will the three sounds happen at the same time (up to 48 beats)?

4. When is the first time that the two sounds happen at the same time?

5. Explain the patterns in the beats where multiple sounds are happening at the same time using the language of multiples and common multiples.

Name: ________ Date: ________ Period: ________

Activity 2 Least Common Multiple

1. The "least common multiple" of 6 and 8 is 24. What do you think the term *least common multiple* means?

2. Determine *all* the multiples of 10 and 8 that are less than 100. Then identify the least common multiple.

3. What is the least common multiple of 7 and 9?

Are you ready for more?

1. For two given numbers, can the least common multiple be one of the two numbers? Show or explain your thinking, and provide an example.

2. Can the greatest common factor be one of the two numbers? Show or explain your thinking, and provide an example.

3. Can the least common multiple and the greatest common factor of two different numbers be the same? Show or explain your thinking, and provide an example.

STOP

Summary

In today's lesson . . .

You reviewed that a *multiple* of a whole number is the product of that whole number and another whole number. Given any two whole numbers, you reasoned that you could determine their ***common multiples*** and their ***least common multiple*** (LCM).

Two whole numbers have infinite common multiples, but will only ever have one LCM.

Numbers	Multiples	Least common multiple
4	4 8 12 16 20 24 28 32 36 ...	12
6	6 12 18 24 30 36 42 48 54 ...	
2	2 4 6 8 10 12 14 16 18 ...	8
8	8 16 24 32 40 48 56 48 54 ...	

Reflect:

Name: .... Date: .... Period: ....

Practice

1. Consider different-colored lights that each blink at certain intervals of seconds.

   a. A green light blinks every 4 seconds and a yellow light blinks every 5 seconds. When will both lights blink at the same time?

   b. A red light blinks every 12 seconds and a blue light blinks every 9 seconds. When will both lights blink at the same time?

   c. Explain how to determine when any two lights will blink at the same time.

2. Think about the multiples of 10 and 15.

   a. List *all* the multiples of 10, up to 100.

   b. List *all* the multiples of 15, up to 100.

   c. What is the least common multiple of 10 and 15?

3. At a local store, cups are sold in packages of 8. Napkins are sold in packages of 12.

   a. What is the fewest number of packages of cups and the fewest number of packages of napkins that can be purchased so there will be the same number of cups as napkins?

   b. How many sets of individual cups and napkins will there be?

Practice

Name: ________ Date: ________ Period: ________

4. One batch of light green paint uses 2 cups of green paint and 7 cups of white paint. Jada made a large amount of light green paint by using 10 cups of green paint.

   a. The amount of light green paint she made is equivalent to how many batches?

   b. How many cups of white paint did she use?

5. What are *three* different ratios that are equivalent to the ratio $3 : 12$? Explain how you know your ratios are equivalent.

6. Think about the number 12.

   a. What are *all* the factors of 12?

   b. What are some multiples of 12?

Name: .... Date: .... Period: ....

Unit 2 | Lesson 11

Navigating a Table of Equivalent Ratios

Let's use a table of equivalent ratios.

Warm-up Number Talk

Mentally determine each product.

1. $\frac{1}{3} \cdot 21$

2. $\frac{1}{6} \cdot 21$

3. $5.6 \cdot \frac{1}{8}$

4. $\frac{1}{4} \cdot 5.6$

Log in to Amplify Math to complete this lesson online.

Activity 1 Concert Ticket Prices

Noah's and Jada's families purchased tickets to a Chicago Sinfonietta concert. Complete the table to help with your thinking as you complete the problems.

Number of tickets	Price ($)

1. Noah bought 4 tickets and paid $\$103$. What was the cost per ticket? Show or explain your thinking.

2. Jada's family bought 9 tickets for her family to attend and paid $\$231.75$. Did Jada and Noah pay the same price per ticket? If not, who paid more? Show or explain your thinking.

3. How much would Jada have spent in total if she bought 1 more ticket for the same price?

Name: ______ Date: ______ Period: ______

Activity 2 Chicago Deep-Dish Pizza

After attending the concert, Jada's family heads to a local restaurant to enjoy some famous Chicago deep dish pizza. The crust for an extra large pizza uses 12 tbsp of cornmeal and 16 tbsp of butter. Noah's family is going to make their own deep dish pizza at home, but they don't need nearly as large of a pie.

1. How many tablespoons of cornmeal and butter are needed for the smallest pizza with a crust that has the same consistency and tastes the same, but that can be made using whole numbers of tablespoons of each ingredient? Consider completing the table to help organize your thinking.

Cornmeal (tbsp)	Butter (tbsp)

2. How many other sizes of pizza that are smaller than the restaurant's extra large size can be made using only whole tablespoons of both of those ingredients? What are the ratios of cornmeal to butter?

3. What are the two ratios containing a 1 of cornmeal and butter in the crust recipe?

a Ratio of cornmeal to butter for 1 tbsp cornmeal:

b Ratio of cornmeal to butter for 1 tbsp butter:

STOP

Summary

In today's lesson . . .

You used a ratio table to determine some special equivalent ratios that you have seen before, but now in the context of different scenarios.

One such type of special ratio is when the value for one of the two quantities is equal to 1. These ratios tell you the exact amount of a quantity that corresponds to precisely 1 unit of another quantity. You can see in the Granola-to-Price table that there is a ratio for the cost of 1 pound of granola or the amount of granola for $\$1$. This is often read as the "price *per* pound," or the "unit price," because the word *per* means "for each," or, more specifically, "for each 1."

Another type of special ratio is when the values share a common factor. An equivalent ratio can always be determined by dividing both quantities by the greatest common factor of the numbers in any equivalent ratio. From the table, $16 : 20$ shares the factor of 4.

These types of special ratios are useful for generating equivalent ratios in a ratio table.

	Granola (lb)	Price ($)	
$\div 4$	16	20.00	$\div 4$
$\times \frac{1}{5}$	4	5.00	$\times \frac{1}{5}$
	1	1.25	
$\times 3$	0.8	1.00	$\times 3$
	2.4	3.00	
	62	77.50	

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

1. Kiran reads 22 pages in 44 minutes. He spends the same amount of time per page. Consider using the table to help with your thinking as you solve each of the following problems.

a. How long does it take Kiran to read 1 page?

b. How many pages can he read in 1 minute?

Time in minutes	Number of pages
44	22

2. Mai is making personal pizzas. For 4 pizzas, she uses 10 oz of cheese.

a. How much cheese does Mai use for each pizza?

b. At this same rate, how much cheese will she need to make 10 pizzas?

Number of pizzas	Ounces of cheese
4	10

3. A triple batch recipe of peanut butter granola bars contains 3 cups of peanut butter and 6 cups of oats. How much peanut butter and oats are in one batch of granola bars? Explain or show your thinking.

Practice

Name: .... Date: .... Period: ....

4. Each of these is a pair of equivalent ratios. For each pair, explain how you know they are equivalent ratios.

   a. $600 : 450$ and $60 : 45$

   b. $60 : 45$ and $4 : 3$

   c. $600 : 450$ and $4 : 3$

5. Complete the table to show the amounts of flour and milk needed for a pancake recipe in 3 different-sized batches.

Flour (cups)	Milk (cups)
4	2

6. Plot the following numbers on the number line: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{6}$. Explain how you determined where to plot them.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 12

Tables and Double Number Line Diagrams

Let's use double number lines to represent equivalent ratios.

Warm-up Constant Dividend

1. Mentally determine each quotient.

   a. $150 \div 2$

   b. $150 \div 4$

   c. $150 \div 8$

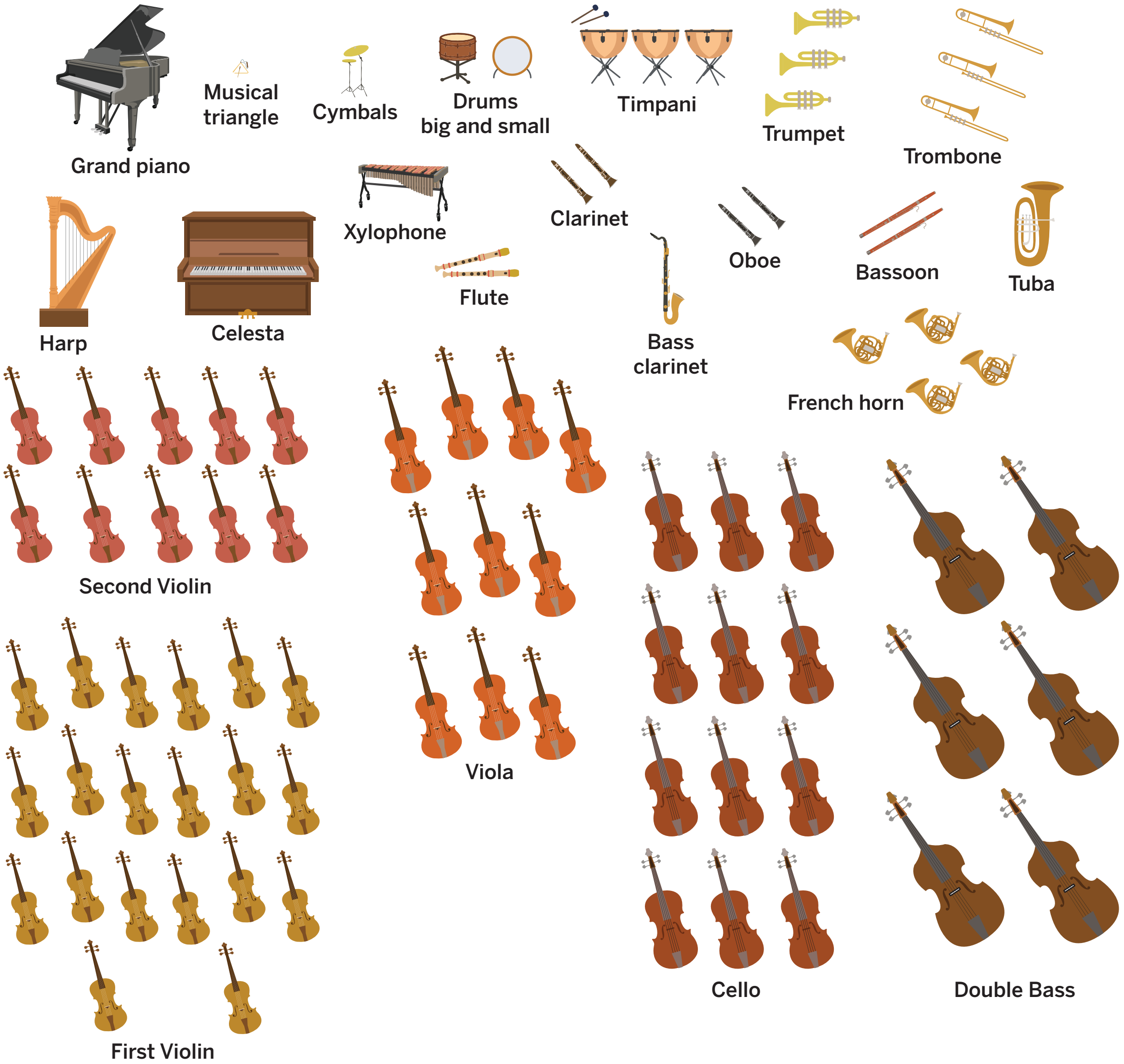
2. Locate and label *all* of the quotients from Problem 1 on this number line.

Log in to Amplify Math to complete this lesson online.

Activity 1 A Larger Orchestra

In 2019, a new Guinness World Record for the largest orchestra was set when 8,097 musicians came together in Saint Petersburg, Russia. When multiple orchestras get together to create a larger orchestra, they want to keep the ratios of instruments equivalent, so the overall sound is balanced in the same ways.

Most orchestras consist of at least 90 instruments, and this image shows the number of each type of instrument in an example orchestra.



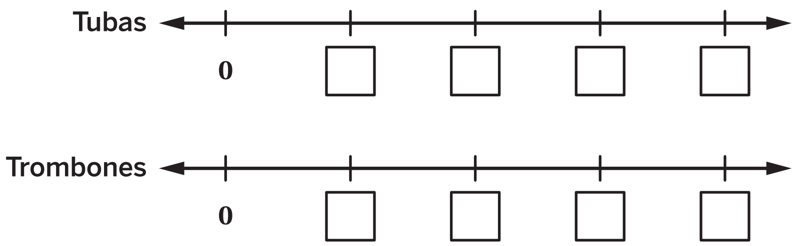
Name: .... Date: .... Period: ....

Activity 1 A Larger Orchestra (continued)

Suppose you need to organize larger orchestras that have the same balance of sound by keeping the ratios of instruments equivalent to those in a typical orchestra.

1. Consider the balance between the tubas and trombones.

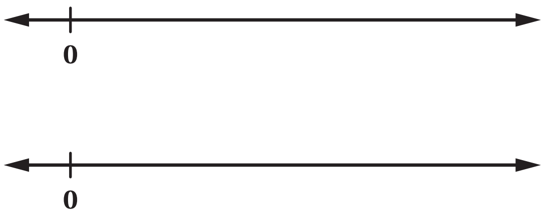
a Complete the double number line to show possible numbers of tubas and trombones in different-sized orchestras.



b Choose two of your ratios and explain how you know they are equivalent.

2. Consider the balance between the French horns and double basses.

a Complete the double number line to show possible numbers of French horns and double basses in different-sized orchestras.



b Choose two of your ratios and explain how you know they are equivalent.

c For each ratio you chose in part b, determine how many total instruments would be in each of those orchestras. Explain or show your thinking.

Activity 2 Tables and Double Number Lines

You want to create other possible numbers of these instruments in larger orchestras, making sure that the ratios are equivalent so that the overall sound stays balanced and sounds the same.

1. Choose two pairs of two different instruments from a typical orchestra.

Instrument pair 1: .... and ....

Instrument pair 2: .... and ....

2. You will create a double number line for one pair of instruments and a table for the other. Your partner will create the opposite representations for the opposite pairs of instruments.

a Create your double number line and table here. Label each representation clearly to show which pair of instruments corresponds to each.

Name: .... Date: .... Period: ....

Activity 2 Tables and Double Number Lines (continued)

b Discuss both representations for both pairs of instruments with your partner. Once you agree on all of the information being presented, modify your representations as needed and copy your partner's representations here.

3. Compare and contrast the pros and cons of using a table versus a double number line to determine and represent equivalent ratios.

Historical Moment

Friendly Numbers

Pairs of related numbers have all sorts of fun names! There are amicable numbers (identified even earlier than 850 CE, and notably worked on by the Iraqi mathematician Thābit ibn Qurra). There are friendly numbers — the ratio of the sum of the factors of each number to itself, called its abundancy index, is the same for both numbers. And then of course there are solitary numbers — basically, not friendly numbers, meaning no other number shares its ratio.

For some numbers, it is not yet known whether they are friendly or solitary. As of 2021, the smallest, and arguably craziest, example is 10. A group of students at Clarkson University published a paper in 2006 that partly proved what must and must not be true about a friend of 10, if it exists. No computer program has found a friend of 10 just yet, but we know any friend must have at least 31 digits!

1. Show that 6 and 28 are a friendly pair.

2. What is the abundancy index of 10, written as a ratio?

STOP

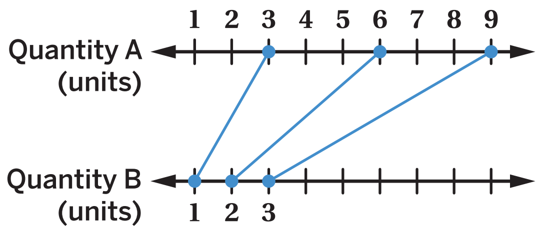
Summary

In today's lesson . . .

You saw a new way to represent equivalent ratios, using *double number line diagrams*.

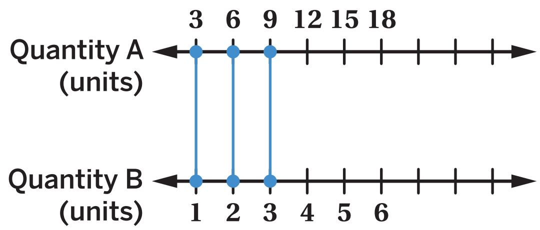
You can choose the scales for the number lines in either of two ways:

Using the *same scale* (such as by 1s).



This is helpful for seeing how much more of one quantity there is than the other, and how the pattern grows.

Using *different scales* (such as by 1s and 3s).



This is helpful for identifying equivalent ratios at a glance.

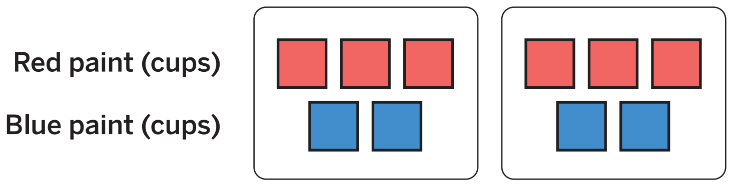
Double number lines and tables are two different representations that can both be used to help generate and identify equivalent ratios. In both representations, you should include labels and units for each quantity. On a double number line, the numbers are always listed in order. In a table, you can write the equivalent ratios in any order.

Reflect:

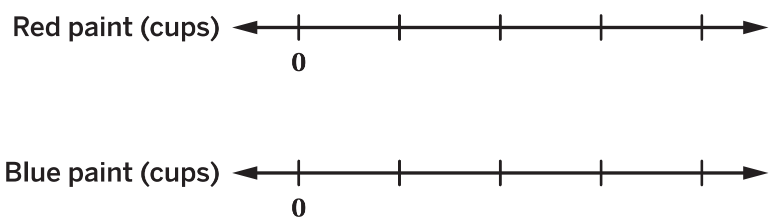
Name: .... Date: .... Period: ....

1. In an orchestra, the ratio of clarinets to cello is $2 : 12$. Multiple orchestras are planning to combine to create a larger orchestra and they want to keep the ratio of instruments equivalent so the sound is balanced the same. Create a table to show how many instruments would be needed in three other possible sizes of orchestras.

2. The diagram shows the amounts of red and blue paint that make 2 batches of a purple paint.



a Complete the double number line representing the amounts of red and blue needed to make the same purple paint. Label the tick marks to show the different amounts of red and blue paint that can be used to make different total amounts of this same purple paint. Equivalent ratios should be aligned vertically.



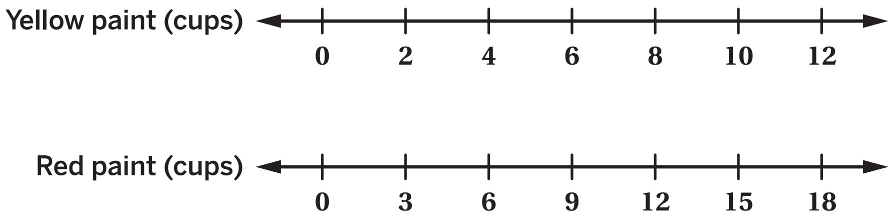
b Making the same shade of purple paint by using 12 cups of red paint is equivalent to making how many batches?

c Making the same shade of purple paint by using 6 cups of blue paint is equivalent to making how many batches?

Practice

Name: .... Date: .... Period: ....

3. One batch of a particular orange paint is made with 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of this same shade of orange paint.



4. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select *all* the statements that represent this ratio.

A. The ratio of kids to pizzas is 7 : 3.

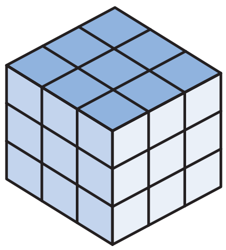
B. The ratio of pizzas to kids is 3 to 7.

C. The ratio of kids to pizzas is 3 : 7.

D. The ratio of pizzas to kids is 7 to 3.

E. For every 7 kids, there needs to be 3 pizzas.

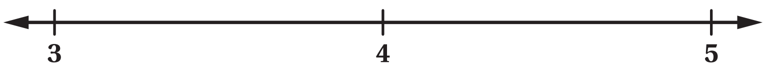
5. In this cube, each small square has a side length of 1 unit.



a What is the surface area of this cube?

b What is the volume of this cube?

6. Plot $3\frac{1}{4}$ and 4.75 at their correct locations on the number line.



Name: ________ Date: ________ Period: ________

Unit 2 | Lesson 13

Tempo and Double Number Lines

Let's look at song tempos and draw more double number line diagrams.

Warm-up Ordering on a Number Line

1. Locate and label the following numbers on the number line.

$\frac{1}{2}$	$1\frac{3}{4}$	$\frac{1}{4}$	1.5	1.75

2. Write one fraction and one decimal that are not equivalent to each other or to any of the numbers plotted on the number line. Plot and label your two numbers on the number line.

Log in to Amplify Math to complete this lesson online.

Activity 1 Song Tempos

Tempo **is the speed or pace at which a song is played. In Western classical music, Italian words are used to describe different tempo markings, which correspond to different ranges of beats per minute (bpm). These tempo markings can also be used to describe how to dance to such a song. Refer to the table of different tempo markings and their corresponding beats per minute.**

Tempo marking	Common definition (bpm)
Prestissimo	Very very fast (> 200 bpm)
Presto	Very fast (169–200 bpm)
Allegro	Fast (121–168 bpm)
Moderato	Moderate (109–120 bpm)
Andante	Walking pace (76–108 bpm)
Adagio	Slow and stately (66–75 bpm)
Lento/Largo	Very slow (41–65 bpm)
Grave (grah•vey)	Slow and solemn (20–40 bpm)

1. Think of two songs that you know. One should be a faster song and one should be a slower song. Mark where you think their tempos would be on the line.

2. Assuming all of the songs described are played at the same tempo throughout, determine the tempo marking of each song.

a A 5-minute song containing 750 beats.

b A 5-minute song containing 225 beats.

c A 3-minute song containing 276 beats.

d A 4-minute song containing 460 beats.

Name: .... Date: .... Period: ....

Activity 1 Song Tempos (continued)

3. Choose a tempo marking from the table and number of minutes for the length of a song.

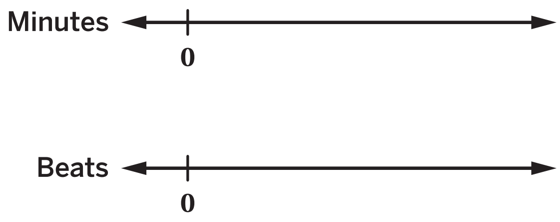
   Tempo: ....

   Length of song (minutes): ....

   a. How many beats per minute could the song have?

   b. Using your answer from Problem 3a, how many total beats would the song have?

   c. Complete the double number line to show the beats for each passing minute.



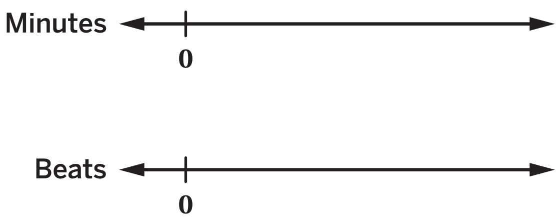
Stronger and Clearer:
After you complete Problem 3, your teacher will provide you some time to work with your partner to clarify and revise your thinking.

Activity 2 Faster and Slower Tempos

1. Frederick Chopin's Waltz no. 10 in B minor is played at a moderato tempo.

a What could be a possible number of beats per minute for this song?

b The song is 3 minutes and 30 seconds long. Complete the double number line to show the number of beats for *each passing minute*.



2. Choose another song from the list with a different tempo than in Problem 1.

a Write your chosen song title here.

b What could be a possible number of beats per minute for this song?

c Create a double number line showing the number of beats for *each passing 30 seconds* of the song (or up to 5 minutes).

3. Which song is being played at a faster tempo? How do your double number lines for Problems 1 and 2 show this?

STOP

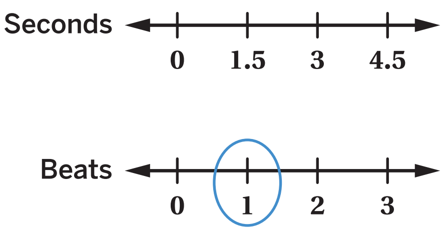
Name: ______ Date: ______ Period: ______

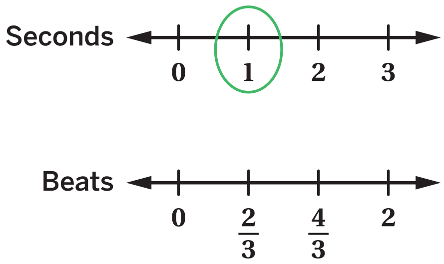
Summary

In today's lesson . . .

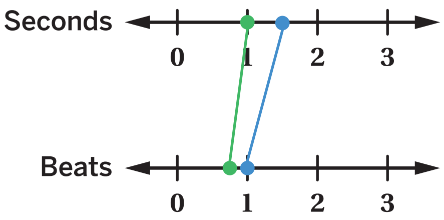
You saw that different songs are played at different tempos, which can be represented as the ratio of beats to seconds.

You can represent the ratio for 1 beat and the ratio for 1 second using two number line diagrams:





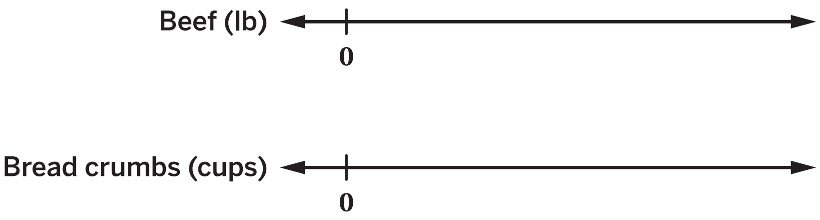
This can also be done with one diagram, using the same scale of 1 on both number lines:



Reflect:

Name: .... Date: .... Period: ....

1. One batch of meatloaf contains 2 lb of beef and $\frac{1}{2}$ cup of breadcrumbs. Complete the double number line to show the amounts of beef and breadcrumbs needed for 1, 2, 3, and 4 batches of meatloaf.



2. The song *Perfect* by Ed Sheeran is 4 minutes and 23 seconds long and is played at a lento/largo 63 bpm. Create a double number line to show the number of beats for each passing minute up to 4 minutes.

3. A recipe for tropical fruit punch says, "Combine 4 cups of pineapple juice with 5 cups of orange juice."

   a. Create a double number to show the amount of each type of juice in 1, 2, 3, and 4 batches of the recipe.

   b. The recipe also calls for $\frac{1}{3}$ cups of lime juice for every 5 cups of orange juice. Add a third number line to your diagram to represent the amount of lime juice in each number of batches of tropical fruit punch.

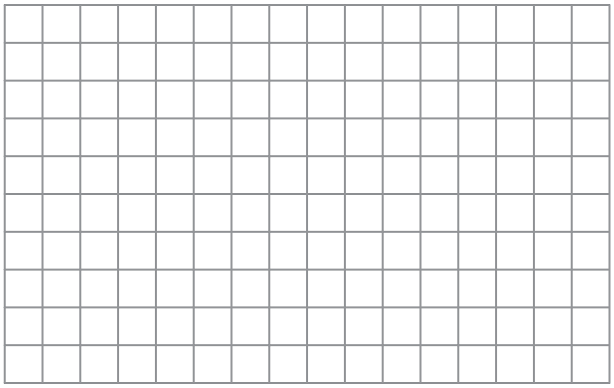
   c. If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

Name: ________________ Date: ________ Period: ________

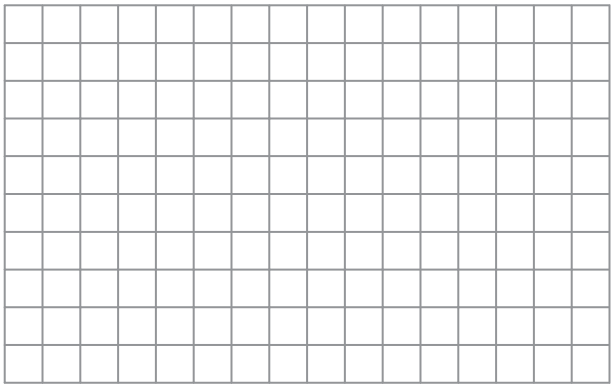
4. Determine three different ratios that are equivalent to $3 : 11$. Explain how you know all of your ratios are equivalent.

5. Draw each of the indicated figures using the grids shown.

a Draw a parallelogram that has an area of 24 square units, but is not a rectangle. Explain or show how you know the area is 24 square units.



b Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



6. Noah bought 7 boxes of pasta. Each box was 12 in. tall. How many pounds of pasta did Noah buy in all?

a What given information do you need to use to solve the problem?

b What given information do you *not* need to use to solve the problem?

c What information is missing that you would need to know in order to solve the problem?

My Notes:

Who brought Italy to India and back again?

In the 1980s, Italian cuisine was rare in Kolkata, India. And yet, for 10-year-old Ritu Dalmia, there was nothing better. She had gotten a taste for it after a school trip to Italy. For a month, she and her classmates sampled dishes like spaghetti pomodoro. For Dalmia, it was love-at-first-taste.

This love would start her on a journey many decades long, spanning multiple countries.

She opened MezzaLuna, one of Delhi's first Italian restaurants. Two years later, Dalmia headed to London to open Vama, a successful, high-end Indian restaurant. Five years after that, she returned to India to open another Italian restaurant — Diva. Diva was so successful that offshoots sprouted up, including Diva Cafe, DIVA Piccola, and Latitude 28. Not one to rest on her laurels, Dalmia returned to the source — Italy — to open Cittamani. This exciting new restaurant fused Indian cuisine with Italian ingredients.

Dalmia's passion has brought new tastes and flavors to those who might not otherwise have the opportunity to try them. Whether you're a home cook or a globe-hopping celebrity chef, the right ingredients in the right amounts are important to executing a meal. But to get the recipe exactly right, ratios are the key ingredient!

Unit 2 | Lesson 14

Solving Equivalent Ratio Problems

Let's practice identifying needed information to solve ratio problems.

Warm-up What Do You Want to Know?

You know that a red car and a blue car both entered the same highway at the same time and both have been traveling at constant speeds. You want to know how far apart they are after 4 hours.

What information would you need to know in order to determine how far apart they are after 4 hours? Be prepared to explain *why* you need that information.

Log in to Amplify Math to complete this lesson online.

Name: ______________________ Date: __________ Period: __________

Activity 1 Info Gap: Selling Hot Chocolate

Jada and Noah are going to sell hot chocolate in the cafeteria during lunch. Noah will make the hot chocolate, and Jada will make the signs to advertise their new business.

You will receive either a *problem card* or a *data card*. Do not show or read your card to your partner.

If you are given the *problem card*:	If you are given the *data card*:
1. Silently read your card and think about what information you need to be able to solve the problem.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner "What specific information do you need?" and wait for them to *ask* for information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask "Why do you need that information?" Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently, using a representation of your choice.	**4.** Read the *problem card* and solve the problem independently, using a representation of your choice.
5. Read the *data card*. Share your representation, and discuss your thinking.	**5.** Share the *data card* and your representation, and discuss your thinking.

Are you ready for more?

Noah accidentally added 5 tbsp of cocoa to the hot chocolate mix, and the ratio of cocoa to milk is now $9 : 11$. How many tablespoons of cocoa and cups of milk were in the original mix?

STOP

Summary

In today's lesson . . .

You solved problems involving equivalent ratios by using three given pieces of information:

- Two values that allow you to write a ratio describing the relationship between the two quantities involved.
- A third value that gives a different amount of one of the quantities, which indicates you are interested in determining a corresponding fourth value to make an equivalent ratio.

Suppose you wanted to determine the missing value in the given ratio table, there are multiple methods to consider:

Time	Length
5	2
?	16

$\times 8$ $\times 8$

To go from 2 to 16 multiply by 8.

$5 \cdot 8 = ?$

The missing value is 40.

Time	Length
5	2
?	16

To go from 2 to 5 multiply by $\frac{5}{2}$.

$16 \cdot \frac{5}{2} = ?$

The missing value is 40

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for $\$3.00$, but allows customers to buy any amount of vinegar. Which of the following ratios correctly represents the price of the vinegar? Select *all* that apply.

A. 4 gallons to $\$3.00$

B. 1 gallon to $\$1.50$

C. 30 gallons to $\$45.00$

D. $\$2.00$ to 30 gallons

E. $\$1.00$ to $\frac{2}{3}$ gallons

2. A caterer needs to buy 21 lb of pasta for a wedding. A local store sells handmade pasta by the pound. It costs $\$12$ for 8 lb of pasta. Consider this question: If all pasta is sold at this same rate, how much will the caterer pay for the pasta they need?

a Write a ratio for the given information about the cost of pasta.

b To answer the question, would it be more helpful to write an equivalent ratio using 1 lb of pasta or $\$1$? Explain your thinking, and then write that equivalent ratio.

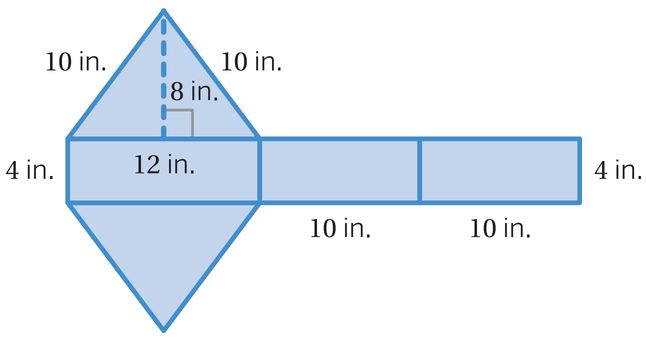
c Calculate the answer to the question and show or explain your thinking.

3. Lin is reading a 47-page book. She read the first 20 pages in 35 minutes. If she continues to read at the same rate, will she be able to complete this book in less than 1 hour? Show or explain your thinking.

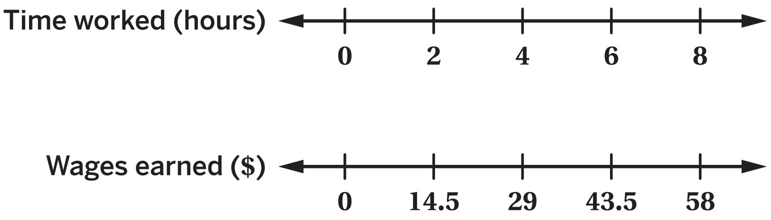
Practice

Name: ________ Date: ________ Period: ________

4. Determine the surface area of the polyhedron that can be assembled from this net. Show your thinking.



5. A cashier worked an 8-hour day and earned $58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your thinking.



a How much does the cashier earn per hour?

b How much will the cashier earn if she works 3 hours?

6. An art teacher is making three different mixtures of orange paint. Identify the mixture, or mixtures, that satisfy each condition.

- **Mixture A:** 4 ml red paint and 3 ml yellow paint
- **Mixture B:** 4 ml red paint and 2 ml yellow paint
- **Mixture C:** 5 ml red paint and 3 ml yellow paint

a The most amount of paint is made.

b The most amount of yellow paint is used.

c The paint that looks the most yellow.

Name: .... Date: .... Period: ....

Unit 2 | Lesson 15

Part-Part-Whole Ratios

Let's look at situations where you can add the quantities in a ratio together.

Warm-up Sparkling Orange Juice

Do you love fizzy drinks, but want to limit the amount of artificial sugar you drink? Are you bored with plain old juice for breakfast? Try this easy recipe! To make sparkling juice, mix 4 parts juice with 3 parts soda water.

Write as many ratios as you can that involve orange juice, soda water, and total sparkling orange juice. Include units in your ratios, and be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Activity 1 Making All-Natural Food Coloring

Did you know that you can create all-natural food coloring by using spices, fruits, and vegetables? Not only can these foods be used to create a beautiful array of colors, they also provide a boost of vitamins and nutrients to any recipe. Note: While spices can be mixed directly with water, fruits and vegetables should first be pressed and juiced to avoid having skins in the final mixture.

1. A recipe for purple food coloring calls for 5 tsp of blueberry juice and 3 tsp of water.

a How many teaspoons of food coloring would this recipe make?

b Shawn needs a batch of 32 tsp of food coloring. How much of each ingredient should Shawn use? Show or explain your thinking.

c How *many times* smaller is the original recipe than Shawn's batch?

2. A red food coloring recipe says, "Mix 4 tbsp raspberry juice with 3 tbsp of strawberry juice and 2 tbsp of water." Kiran wants to make 45 tbsp of red food coloring. He has plenty of water, but he only has 24 tbsp of raspberry juice and 21 tbsp of strawberry juice.

a Does Kiran have enough ingredients to make 45 tbsp of red food coloring? Show or explain your thinking.

Name: ______ Date: ______ Period: ______

Activity 1 Making All-Natural Food Coloring (continued)

b What is the most food coloring Kiran can make using the ingredients he has? Show or explain your thinking.

Are you ready for more?

Use all of the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

☐ : ☐ **is equivalent to** ☐ ☐ : ☐ **and** ☐ ☐ : ☐ ☐

Activity 2 Buying Supplies

Han is excited to experiment with flavored sparkling water and natural food colorings. He wants to buy some supplies by using the nickels, dimes, and quarters he has saved up in his piggy bank. For every 2 nickels, there are 3 dimes. For every 2 dimes, there are 5 quarters. There are 500 coins total. How much money does Han have to buy supplies? Show or explain your thinking.

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You worked with ratios that describe a relationship among two or more quantities that have the same units and can be combined (or added together) to make a *total* amount of some other quantity. The total can also be represented in ratios, and you can use equivalent ratios to solve problems with one *or more* unknown quantities.

For example, mixing 3 cups of yellow paint with 2 cups of blue paint produces a total of 5 cups of green paint. If you need to make 15 cups of green paint, you can use the ratio of $3:2:5$ for blue to yellow to green (total) paint to determine how much yellow *and* blue are needed.

Yellow (cups)	Blue (cups)	Green (cups)
3	2	5
?	?	15

$\times 3$ $\times 3$

$3 \cdot 3 = ?$
9 cups of yellow paint

$2 \cdot 3 = ?$
6 cups of blue paint

Ratios can also represent relationships among quantities when the specific units are not known. For example, 3 parts of yellow paint for every 2 parts of blue paint will still produce 5 parts of the same green paint. Any appropriate unit, such as teaspoons or cups or gallons, can be used in place of "parts" without changing the ratio of $3:2$.

Reflect:

Practice

Name: Date: Period:

1. The ratio of cats to dogs at a boarding facility one weekend is $4 : 5$. There are 27 dogs and cats staying for the weekend in all. How many dogs are there? How many cats are there? Show your thinking.

2. Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Show or explain your thinking.

3. A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. If the teacher orders 85 tickets, how many tickets are for chaperones, and how many are for students? Show or explain your thinking.

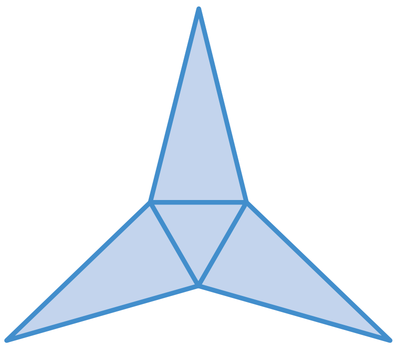
4. In a triple batch of a spice mix, there are 6 tsp of garlic powder and 15 tsp of salt.

a How much garlic powder should be used to make the same spice mix with 5 tsp of salt?

b How much salt should be used to make the same spice mix with 8 tsp of garlic powder?

c If there are 14 tsp of this spice mix, how much salt is in it?

5. Which type of polyhedron can be assembled from this net?



A. Triangular pyramid

B. Trapezoidal prism

C. Rectangular pyramid

D. Triangular prism

6. Usain Bolt is a Jamaican sprinter who won gold medals in three consecutive Olympic Games. His top speed has been measured at 27 miles per hour. Select *all* of the animals whose top speed is *slower* than Usain Bolt's.

A. Elephant: 15 miles per hour

B. Lion: 25 miles per half hour

C. Squirrel: 3 miles per 20 minutes

D. Roadrunner: 5 miles per 15 minutes

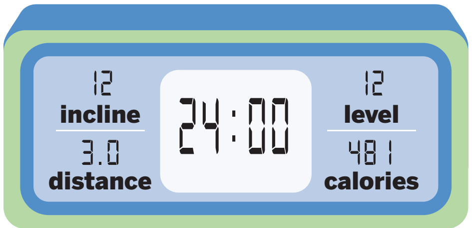
Comparing Ratios

Let's compare ratios.

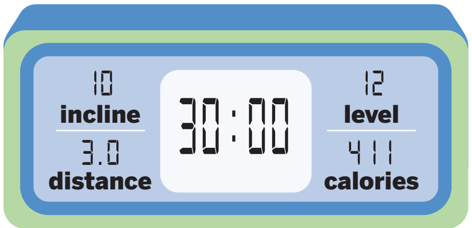
Warm-up Notice and Wonder

Mai and Jada each completed a run on a treadmill. Their treadmill displays are shown. What do you notice? What do you wonder?

Mai's treadmill display.



Jada's treadmill display.



1. I notice . . .

2. I wonder . . .

Log in to Amplify Math to complete this lesson online.

Name: ____________ Date: ____________ Period: ____________

Activity 1 Comparing Chili Peppers

Have you ever taken a bite of a chili pepper and felt like your mouth was on fire? Blame it on the capsaicin (kap·sei·sn), a natural chemical found in most varieties of peppers. The more capsaicin, the spicier the pepper. The level of spiciness is measured on a scale of Scoville Heat Units (SHU). It seems that usually the spicier the pepper, the more expensive it is. In fact, pure capsaicin can measure up to 16,000,000 SHU and can cost as much as $49 for one ounce!

Andre bought ground chili powders of six different kinds of peppers.
He paid:

- $40 for 8 oz of Trinidad Scorpion
- $5 for 4 oz of Jalapeño
- $18 for 2 oz of Carolina Reaper
- $12 for 3 oz of Ghost Pepper
- $20 for 16 oz of Chipotle
- $20 for 10 oz of Habanero

List the six chili powders in order from most to least expensive, and include their unit prices (price per ounce). Show or explain your thinking.

Activity 2 All-Natural Flavoring

You will each design a recipe for your own all-natural flavoring using ingredients from around the world.

- **Your group will be given a set of six flavor cards. Sort the cards into a *sweet* pile and a *sour* pile.**
- **Take turns drawing cards. Each person should select one sweet card and one sour card.**

1. Assign a different number of parts, anywhere from 2 to 20, for each flavor.

a Write the ratio of your ingredients, including the units.

b Describe the flavor of your mix. Be sure to include whether the overall flavor is more sweet or more sour.

2. Compare the sweetness *and* the sourness of the flavor mixtures each member of your group concocted. Show or explain your thinking.

Are you ready for more?

Choose two recipes from your group. How can you make both soda waters taste the same — the same sweetness and sourness? Change as little as possible, and only by adding.

STOP

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You compared scenarios involving ratios by checking if the scenarios represent something happening at the *same rate*. You created an equivalent ratio for one or both scenarios so that the value (and units) for one quantity in each ratio is the same.

For example, let's compare the price for blue and red paints. 6 liters of red paint costs $8, and 2 liters of blue paint cost $3. Which color paint is more expensive (costs more per liter)?

There are multiple methods to consider:

- Making the number of liters the same and comparing the price.
- Making the price the same and comparing the amount of paint.
- Comparing the price for 1 liter for both paints.

Red Paint

Strategy	Liters	Price ($)
Same liters	6	8
Same price	18	24
Unit price	1	1.33

Blue Paint

Strategy	Liters	Price ($)
Same liters	6	9
Same price	16	24
Unit price	1	1.5

- Price for 6 liters is higher for blue paint than for red paint.
- For the same price of $24 you can buy less blue paint than red paint.
- One liter of blue paint costs more than one liter of red paint.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A slug travels 3 cm in 3 seconds. A snail travels 6 cm in 6 seconds. Both travel at constant speeds. Mai says, “The snail was traveling faster because it went a greater distance.” Do you agree with Mai? Show or explain your thinking.

2. If you blend 2 scoops of chocolate frozen yogurt with 1 cup of milk, you will make a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate frozen yogurt with 2 cups of milk. Show or explain why this is true.

3. There are 2 mixtures of light purple paint.
   - Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
   - Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

   Which mixture is a lighter tint of purple? Explain your thinking.

Name: .... Date: .... Period: ....

Practice

4. Diego can type 140 words in 4 minutes.

a. At this same rate, how long will it take him to type 385 words?

b. At this same rate, how many words can he type in 15 minutes?

5. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move a total of 72 boxes?

6. Here are two lemonade recipes.

Recipe A: Mix 3 cups of lemon juice with 2 cups of water.

Recipe B: Mix 3 cups of lemon juice with 3 cups of water.

a. What fraction of Recipe A is lemon juice?

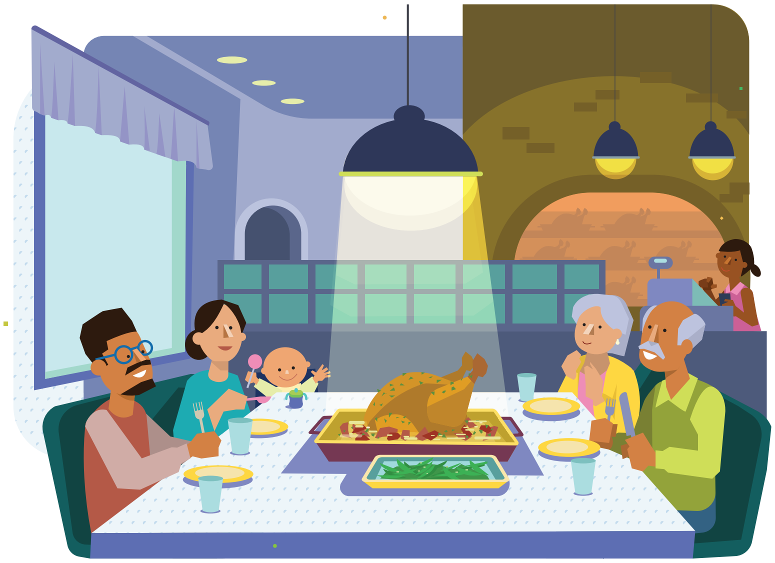
b. What fraction of Recipe B is lemon juice?

c. Which recipe is more tart (stronger lemon flavor)? Explain your thinking.

Unit 2 | Lesson 17

More Comparing and Solving

Let's practice using ratios to solve more problems.



Warm-up Is First Always Fastest?

Tyler and Shawn wanted to finish their books today. They both started reading at the same time. Tyler finished the last 5 pages of his book in 10 minutes. Shawn finished the last 10 pages in 15 minutes.

1. Who finished their book first? Be prepared to explain your thinking.

2. Who read the fastest? Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Catering an Event

A local restaurant is catering a large event. The main course is baked chicken with a garlic parmesan sauce, roasted red potatoes, and sautéed green beans. Here is the progress of each sous chef.

- **Lin needed to prepare 108 cups of garlic parmesan sauce. She has made the first 54 cups in 3 hours.**
- **Diego roasted the first 100 cups of red potatoes in 4 hours. He still needs to prepare 45 more cups.**
- **Clare sautéed 160 cups of green beans in 5 hours. She needs to prepare a total of 192 cups.**

1. If each sous chef started at the same time and continues to work at these same constant rates, in what order will the dishes be completed? Show or explain your thinking.

2. Order how quickly the sous chefs worked from fastest to slowest. Explain your thinking.

Fastest **Slowest**

Activity 2 The Bliss Point

Have you ever wondered why you crave certain foods? Well, ratios (and science) can explain that. Foods are most desirable when they hit the *bliss point* — the "just right" ratio of salt to sugar to fat. When combined, these nutrients activate the reward centers in your brain, causing you to want more, more, more!

The test lab of a food company is experimenting with three new flavors of flavored pretzels by altering the salt : sugar : fat ratio in the seasoning mixes.

Recipe	Salt (parts)	Sugar (parts)	Fat (parts)
A	3	2	1
B	3	7	2
C	3	6	1

1. Describe the flavor of each recipe.

2. Order the recipes from most salty to least salty, most sweet to least sweet, and most rich (fat content) to least rich (fat content). Show your thinking.

a

Most salty **Least salty**

b

Most sweet **Least sweet**

c

Most rich **Least rich**

STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You expanded on your understanding of equivalent ratios to compare ratios to determine which is happening at a greater or lesser rate.

For example, consider two recipes for sweet and sour sauce using sweet honey and sour pineapple juice to determine which is more sour. In Recipe A the ratio of honey to pineapple is $5 : 11$, in Recipe B the ratio is $9 : 23$

You can compare the amount of sour pineapple juice to the total amount of parts in each recipe.

Recipe A

Pineapple	Total
11	16
22	32

$\times 2$ $\times 2$

Recipe B

Pineapple	Total
23	32

By making the total equivalent in each recipe, you can see that Recipe B is more sour than Recipe A since $23 > 22$.

Reflect:

Practice

Name: ________ Date: ________ Period: ________

Jada is making hot chocolate. She has 3 different powdered flavor packets that she can add to the milk. Each ingredient is measured in ounces. Use this table to complete Problems 1–3.

Packet	Chocolate	Vanilla	Cinnamon
A	3	2	3
B	3	1	2
C	5	2	5

1. Jada says that Packet C will have the strongest chocolate flavor. Do you agree or disagree? Show or explain your thinking.

2. Compare the flavor of each recipe.

3. Jada mixed several of the same flavor packets together in a bowl. Her mixture has a ratio of chocolate to vanilla to cinnamon of $36 : 12 : 24$.

a. Which flavor packet did she mix together? Explain your thinking.

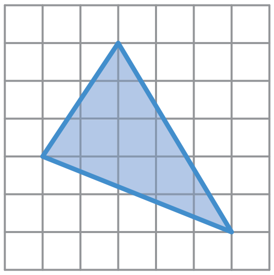
b. How many of the packets did she mix together? Explain your thinking.

Name: ________ Date: ________ Period: ________

Practice

4. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers? Show your thinking.

5. Determine the area of the triangle. Show your thinking.



6. Identify a unit of measurement that can be used to measure:

a The length of a neighborhood road.

b The volume of a car's gas tank.

c The weight of a barbell.

Unit 2 | Lesson 18

Measuring With Different-Sized Units

Let's measure the length, volume, or weight of an object by using different units.

Warm-up Matching Units to Attributes

Write each unit in the appropriate column of the table for the attribute of an object it can be used to measure.

centimeter (cm)	cup (c)	inch (in.)	gram (g)
kilogram (kg)	kilometer (km)	liter (l)	meter (m)
ounce (oz)	pound (lb)	quart (qt)	yard (yd)

Length	Volume	Weight

Log in to Amplify Math to complete this lesson online.

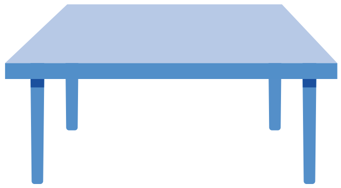
Name: .... Date: .... Period: ....

Activity 1 Units in the Real World

For each of the images shown, write an attribute (length, volume, or weight) you could measure. Then write an appropriate unit of measurement for the chosen attribute. You should use each attribute of length, volume, or weight at least once.

Attribute:

Unit of measurement:



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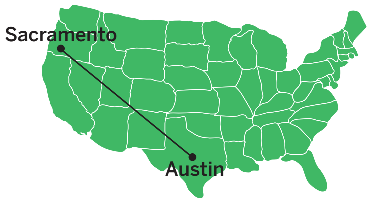

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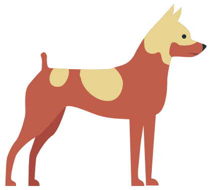

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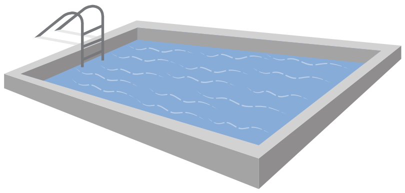


Attribute:

Unit of measurement:

Activity 2 Measurement Stations

Station 1: Length

1. You will be given two different objects to measure. Estimate what you think the measurement would be for each item.

   Item 1: .... in. .... cm

   Item 2: .... in. .... cm

2. Use a ruler to measure each object to the nearest whole number in both inches and centimeters. Record your measurements in the table.

Length of:	Inches	Centimeters

3. Did it take more inches or centimeters to measure the indicated length? Why?

Station 2: Weight

1. You will be given two different objects to measure on the scale. Estimate what you think the weight is of each object.

   Item 1: .... oz .... lb .... g .... kg

   Item 2: .... oz .... lb .... g .... kg

2. Use a scale to weigh each object with as many different units as possible. Record your measurements in the table.

Object	Ounces	Pounds	Grams	Kilograms

Name: ______________________ Date: ____________ Period: ____________

Activity 2 Measurement Stations (continued)

3. Did it take more ounces or grams to weigh the indicated object? Why?

Station 3: Volume

1. Look at the one-gallon jug of water. Estimate how many quart and liter bottles it will fill. Use decimals as needed in your estimates.

   A gallon of water: ____________ quarts ____________ liters

2. You will be given materials to conduct the following experiment (or will watch a video of the experiment) to measure the volume in both quarts and liters. Record your measurements, estimating when necessary, in the table.

   - Empty the gallon of water into the quart bottles, making sure to fill each bottle fully. How many quarts can be filled from the gallon jug? Record your response in the table.
   - Refill the gallon jug and repeat the process of emptying it into the liter bottles. How many liters can be filled from the gallon jug? Estimate to the nearest tenth. Record your response in the table.

	Quarts	Liters
1 gallon		

3. Which is the larger unit, a quart or a liter? Explain your thinking.

STOP

Summary

In today's lesson . . .

You reviewed some standard measurement units for the attributes of length, volume, and weight. By experimenting with everyday objects, you saw that the size of the unit you use to measure something affects the measurement.

If you measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement.

- For example, a room that measures 4 yd in length will also measure 12 ft in length. This makes sense based on the sizes of those two different units because a yard is longer than a foot.
- A similar relationship is true when weighing an object in pounds and then in ounces; or measuring the volume of a container in gallons and then in quarts.

The size of the object relative to the attribute you are measuring, and the amount of precision you need for your measurement, can help you determine the best unit of measurement.

Reflect:

Name: ______________ Date: ______________ Period: ______________

Practice

1. Determine whether each pair of units measures length, volume, or weight and place a check mark in the appropriate column in the table. Then circle or underline the larger unit in each pair.

	Length	Volume	Weight
yard or foot			
quart or gallon			
meter or kilometer			
pound or ounce			
gram or kilogram			

2. Clare says, "This classroom is 11 m long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yd." Do you agree or disagree with Clare? Explain your thinking.

3. Tyler wants to mail a package that weighs $4\frac{1}{2}$ lb. Which of the following could be the weight of the package in kilograms?

A. 2.04 kg

B. 4.5 kg

C. 9.92 kg

D. $4{,}500$ kg

Name: .... Date: .... Period: ....

4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. She wants to make 35 cups of sparkling apple juice for a party. How much of each ingredient should Elena use? Show or explain your thinking.

5. Lin bought 3 hats for $22.50. At this same rate, how many hats could she buy with $60.00? Use the table to help with your thinking.

Number of hats	Price ($)

6. In one minute, Han runs 500 ft and Lin runs 750 ft.

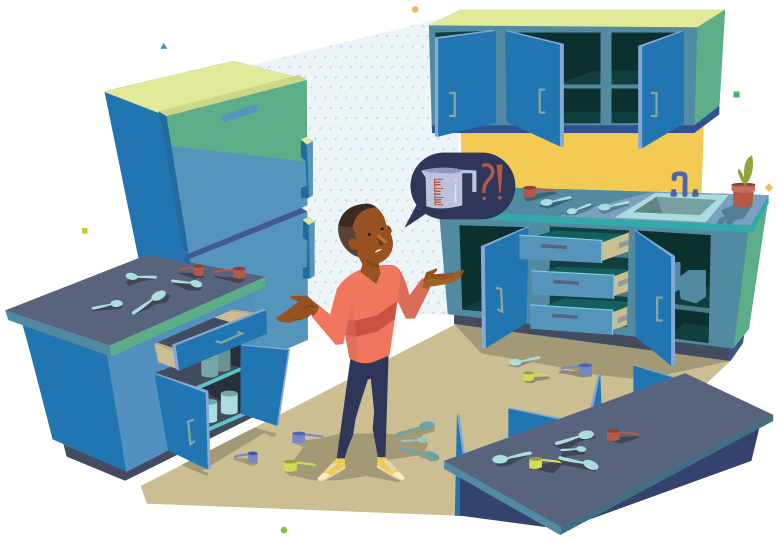
a If they each run at those same rates, how far would each run in 20 minutes?

b In 20 minutes, how many *times farther* does Lin run than Han?

Name: ________________ Date: ________ Period: ________

Unit 2 | Lesson 19

Converting Units



Let's convert measurements to different units.

Warm-up Road Trip

Elena and her mom are visiting England from the United States. One day, Elena's mom was driving on the highway at a speed of 60 miles per hour when she got pulled over by the police for speeding. Outside the car, Elena noticed this road sign.

1. What do you think happened?

2. Where else might Elena and her mom run into similar issues while they are exploring England?

Log in to Amplify Math to complete this lesson online.

Activity 1 Cooking With a Tablespoon

Noah wants to make apple crisp using the following recipe, but he cannot find any measuring cups! He only has a tablespoon (tbsp) for measuring. Luckily, in the cookbook it says that 1 cup is equivalent to 16 tbsp, and 1 tbsp is equivalent to 3 teaspoons (tsp).

Apple crisp recipe
4 medium-size apples, chopped
$\frac{3}{8}$ cup brown sugar
$\frac{3}{4}$ cup oats
$\frac{1}{4}$ cup butter
$\frac{1}{2}$ cup chopped pecans
2 tsp cinnamon
1 tsp vanilla extract

1. Complete the table to help Noah adjust the recipe so that all measurements are in tablespoons.

Apple crisp recipe
4 medium-size apples, chopped
________ tbsp brown sugar
________ tbsp oats
________ tbsp butter
________ tbsp chopped pecans
________ tbsp cinnamon
________ tbsp vanilla extract

Critique and Correct: After you complete Problem 2, your teacher will provide you with an incorrect statement about this situation. Work with your partner to identify and analyze the error(s) and write a correct statement.

2. Noah decides to add in some dried cranberries to the recipe, and measures 10 tbsp. As he updates the original recipe he writes $\frac{2}{3}$ cup of cranberries. Did he write the correct amount? Show or explain your thinking by using a double number line diagram, a table, or other representation.

Name: ________ Date: ________ Period: ________

Activity 2 Metric Recipes

You found a recipe for Chicken and Mushroom Pie online, but all the measurements are in metric units (milliliters and grams). Your measuring cup only shows cups and fractions of cups, and your scale only displays weight in ounces. In order to make the recipe with the tools you have, you need to convert the amounts from metric to U.S. Customary units.

Chicken and Mushroom Pie
25 ml canola oil
420 g skinless boneless chicken thighs
110 g chopped onion
250 g mushrooms
42 g flour
360 ml chicken stock
200 ml milk
1 package of puff pastry
1 egg

Approximate Conversions:
237 ml $\approx$ 1 cup
28 g $\approx$ 1 oz

You will be given two sets of cards:

- One with the amount of each ingredient from the recipe (except the puff pastry and egg) in metric units.
- One with the same amounts converted to U.S. Customary units (but the units have been left off).

1. Work with your partner to match one card from each set for each ingredient using the approximate conversions provided. You may use a calculator to perform the conversions. Round each conversion to the nearest tenth.

2. Complete the table on the next page.
  - Paste or copy the recipe amount in metric units in the first column.
  - Paste or copy the corresponding amount converted into U.S. Customary units in the second column. Be sure to write in the appropriate units: cups or ounces.
  - Explain or show your thinking in the third column.

Activity 2 Metric Recipes (continued)

Recipe amount (metric units)	Converted amount (U.S. Customary units)	Explain or show your thinking:

STOP

Name: ______________________ Date: __________ Period: __________

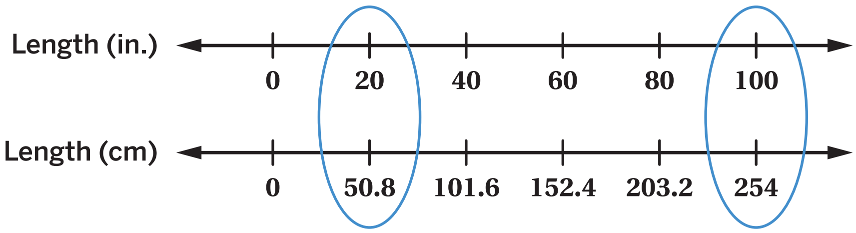
Summary

In today's lesson . . .

You saw that when you measure the same attribute of two or more objects by using the same two different units, the pairs of measurements are equivalent ratios. You can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you measure the side of a table to have a length of 20 in. You want to know this length in centimeters. Given that 100 in. is equal to 254 cm, you can use the ratio of inches to centimeters of 100 : 254 to determine an equivalent ratio for 20 in. This can be done and represented in several ways.

Using a double number line diagram:



Using a ratio box or a table:

	Inches	Centimeters	
	100	254	
$\div 100$	1	2.54	$\div 100$
$\times 20$	20	50.8	$\times 20$

Reflect:

Practice

Name: ______ Date: ______ Period: ______

1. Priya's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Pesos	Dollars
4,250	250
	25
	1
	3
510	

2. There are 3,785 ml in 1 gallon, and there are 4 qt in 1 gallon.

a. How many milliliters are in 3 gallons? Show or explain your thinking.

b. How many milliliters are in 1 quart? Show or explain your thinking.

3. Tyler is making a soup that calls for 28 oz of potatoes. Determine the approximate weight of potatoes needed in both kilograms and grams. **Note:** 1 kg is approximately 35 oz.

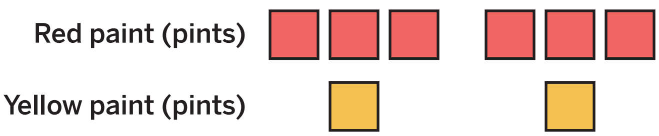
4. A simple trail mix uses only 7 oz of almonds for every 5 oz of raisins. How many ounces of almonds would be in a one-lb bag (16 oz) of this trail mix? Show or explain your thinking.

Name: ________________ Date: ________ Period: ________

5. Identify whether each unit measures length, volume, or weight by placing a check mark in the appropriate column. Then circle the largest unit from each category.

Unit	Length	Volume	Weight
mile			
cup			
pound			
milliliter			
yard			
gram			
kilogram			
pint			
liter			
teaspoon			
centimeter			

6. The diagram represents the pints of red and yellow paint in a mixture. Select *all* statements that accurately describe the diagram.



A. The ratio of yellow paint to red paint is 2 to 6.

B. For every 3 pt of red paint, there is 1 pt of yellow paint.

C. For every pint of yellow paint, there are 3 pt of red paint.

D. For every pint of yellow paint there are 6 pt of red paint.

E. The ratio of red paint to yellow paint is 6 : 2.

More Fermi Problems

Let's solve a Fermi problem.

Warm-up Making Guesses

1. Choose one of these Fermi problems that you would like to answer.
   - How many sticky notes will it take to cover the Washington Monument?
   - How many insect fragments are allowed to be in the 2020 world's largest chocolate bar that weighed approximately 12,770 lb?
   - If a radio station played your favorite song non-stop for the rest of your life, how many times would you hear it?

2. Use the following structure to make your best first guess (without any calculations) for the answer to your problem. Be prepared to explain your thinking.

   a. A number that is probably too small.

   b. A number that is probably too big.

   c. Your best first guess.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Educated Guesses and Calculating

Part 1: Educated Guesses

1. List any questions you would need to answer first in order to arrive at a more likely solution to your chosen Fermi problem from the Warm-up.

2. Write your best guesses for each question on your list.

3. Based on your guesses, what do you think is the answer to your Fermi problem? Show or explain your thinking.

Activity 1 Educated Guesses and Calculating (continued)

Part 2: Gathering Data and Calculating

4. You will be given a data card, or time to conduct your own research. Use the information gathered to first answer the questions on your list more precisely, adding or refining questions as necessary. Then use the information to calculate an answer to your Fermi problem. Show or explain your thinking.

5. Create a poster that will be displayed for a Gallery Tour. Your poster should clearly show your classmates not only the answer you came up with but also how you worked through the Fermi process. Be sure to include:

- The Fermi problem.
- Your first "wild" guesses.
- Your educated guess.
- Assumptions and estimations you made.
- Your calculations.
- One or two sentences stating your final answer and any other conclusions.

Name: .... Date: .... Period: ....

Activity 2 Posing and Answering a Fermi Question

1. Write another Fermi problem related to the same context as the question you answered in Activity 1. Then write down your best first guess.

2. What information from Activity 1 can you use to solve your new question?

3. What new information do you need?

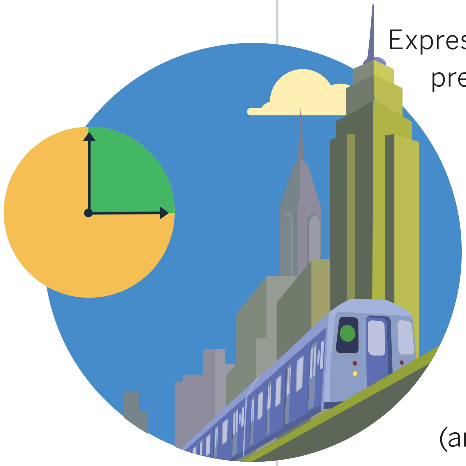
4. Conduct some research to gather the new information you need, and then determine an answer to your Fermi problem. Show or explain your thinking.

STOP

Unit Summary

Ratios are everywhere. They are in the paint on the walls, the food on your plate, even in the rhythms of your music. They give things their consistency and balance. The wrong ratio of red to blue can mean the difference between fuchsia and mauve. A band that's off-rhythm can cause any dancer to stumble.

Much like you saw with Oobleck, the ratios in paint colors, song tempos, and recipes are a constant dance between quantities. In many ways, you've known these quantities all your life. You can taste when soup has too much salt. You can see when the shade is off when mixing colors.



Expressing these ratios mathematically lets you be more precise than with "feel" alone.

While the problems you explored throughout this unit have been light-hearted, your process mirrored that of mathematicians, scientists, business leaders, and policy experts who are working on some of the world's most complex problems. From managing the world's ecosystems and preparing disaster relief efforts to starting a new business and launching spaceships to Mars, ratios play a key role in the world (and Universe) around us.

See you in Unit 3.

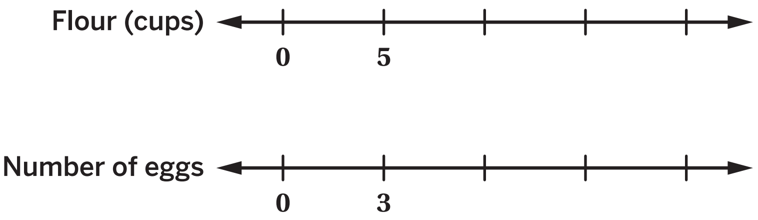
Name: .... Date: .... Period: ....

1. Jada wants to determine how many cups of vanilla pudding it would take to fill an Olympic-sized swimming pool.

a What information does she need to solve this problem?

b How can she use the information to solve the problem?

2. This double number line diagram shows the amount of flour and eggs needed for one batch of almond scones.



a Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of almond scones.

b What is the ratio of cups of flour to eggs?

c How much flour and how many eggs are used in 4 batches of almond scones?

d How much flour is used with 6 eggs?

e How many eggs are used with 15 cups of flour?

3. One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of the same color pink paint using 14 cups of red paint.

a How many batches of pink paint did she make?

b How many cups of white paint did she use?

Practice

Name: .... Date: .... Period: ....

4. Train A travels 30 miles in $\frac{1}{3}$ hour, and Train B travels 20 miles in $\frac{1}{2}$ hour. If both trains travel at a constant speed, explain how you know that Train A is traveling faster than Train B.

5. Diego has 48 strawberry breakfast bars, 64 blueberry breakfast bars, and 100 lemon breakfast bars for a bake sale. He wants to make bags that have all three types of breakfast bars and the same number of each type in each bag.

   a. How many bags can he make without having any breakfast bars left over?

   b. Is there another possible solution? If so, what is another solution?

6. Tyler's height is 57 in. Which of the following could reasonably represent his height in centimeters?

   A. 22.4 cm

   B. 57 cm

   C. 144.8 cm

   D. $3{,}551$ cm

My Notes: