UNIT 2

Dividing Fractions

Division can be used to solve equal-sized groups problems, including when the size of a group and even the number of groups are represented by fractions. See how you can apply what you already know about multiplication and division to follow the mysteries within Spöklik Furniture and fraction division.

Essential Questions

- How can dividing by the same fraction be interpreted in two different ways?
- How is dividing by a fraction related to multiplying fractions?
- What does it mean when a quantity represents a fractional number of equal-sized groups?
- *(By the way, how many tanks could a fish tank fill if a fish tank could fill tanks?)*

SUB-UNIT

1 Interpreting Division Scenarios

Narrative: Find a missing friend at Spöklik Furniture, where not everything is what it seems.

You'll learn . . .

- two ways to think about division.
- connections between multiplication, division, and fractions.

SUB-UNIT

2 Division With Fractions

Narrative: Use fractions to build furniture in the Spöklik Showroom with some ghostly companions.

You'll learn . . .

- strategies for dividing fractions.
- how and why these strategies work.

SUB-UNIT

3 Fractions in Lengths, Areas, and Volumes

Narrative: Make your way out of Spöklik Furniture, and measure with fractions as you go.

You'll learn . . .

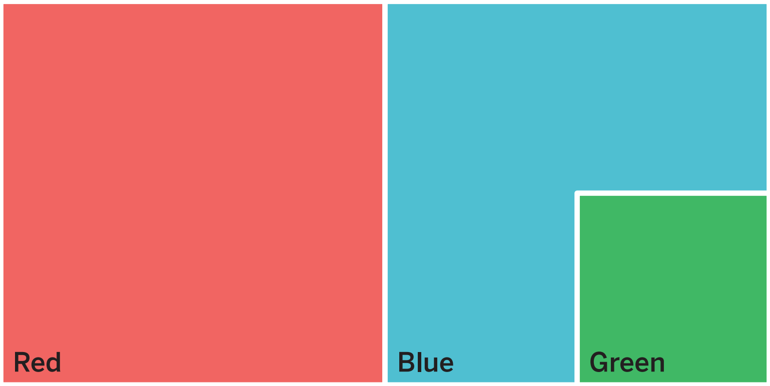
- about fractional measurements.
- to solve measurement problems by dividing fractions.

Seeing Fractions

Let's look for fractions in different patterns.

Warm-up Identifying Fractions

Consider the image. What fractions do you see?



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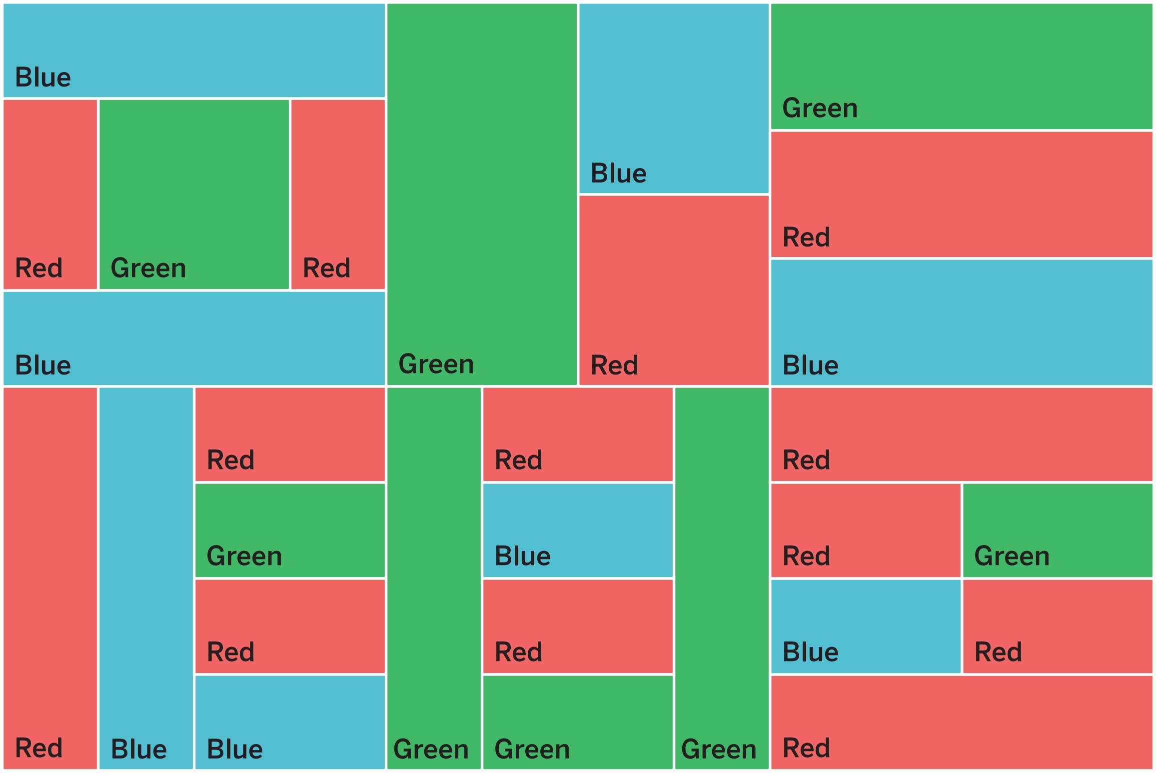
Name: Date: Period:

Activity 1 What Fractions Do You See?

Part 1

Consider the image.



1. You will be assigned one of the two fractions below. How many ways can you see the fraction in the image?

   a. $\frac{1}{12}$

   b. $\frac{1}{8}$

Activity 1 What Fractions Do You See? (continued)

Part 2

2. You will be given a recording sheet. As a group, identify as many fractions and their corresponding wholes as possible before time is called.

3. Give your team 1 point for every fraction on your list. You may count the same fraction more than once as long as it refers to a different whole.

Points: ....

Part 3

4. Compare your list against another group's list. You will earn 1 bonus point for every fraction on *your* list that is *not* on their list. To earn that bonus point, you must explain your thinking to the other group, and they must agree with you.

Bonus points: .... **Total points:** ....

STOP

Reflect: In what ways did you demonstrate confidence as you looked for fractions?

Unit 2 Dividing Fractions

Crossing the Fractional Divide

In most of the units in this course, we talk about the different ways math touches almost every part of our lives — art, history, technology, and current events.

In *this* unit, we'll be talking about fractions — specifically, dividing fractions. You may remember that you use fractions to represent quantities that are not whole numbers, but are instead located *between* whole numbers. But while the idea of fractions might seem intuitive to you, operating with them — adding, subtracting, multiplying, and, yes, dividing them — requires more calculation than whole numbers did. And *understanding* what you are doing at each step often requires careful thought.

But don't worry! With practice and patience, you will get a feel for how to operate with fractions, expanding on the kinds of numbers at your disposal.

With all this mind, let's try something a little different in this unit . . .

Over the next few lessons, you will be reading a story. And, in this story, you will encounter different problems. Some of them will be quite tough, taking your understanding of working with fractions to new depths.

Just remember to relax and breathe. You might want to team up with a friend, or come back to these problems with fresh eyes. With patience and perseverance, you will come out on the other side stronger and more comfortable with dividing fractions.

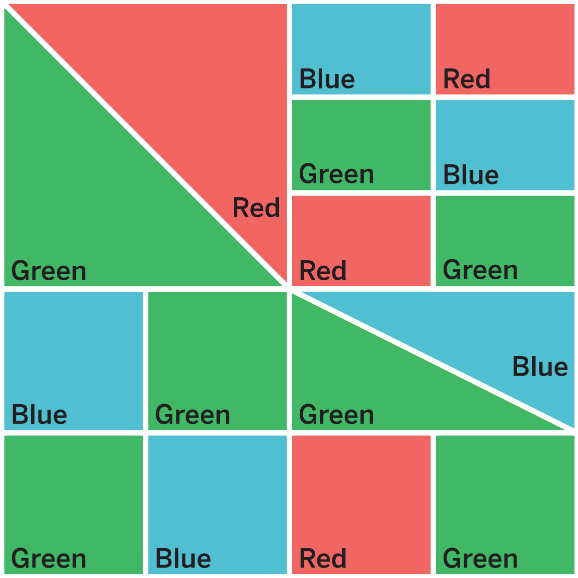
So, when you're ready, turn the page.

Welcome to Unit 2.

Name: ______ Date: ______ Period: ______

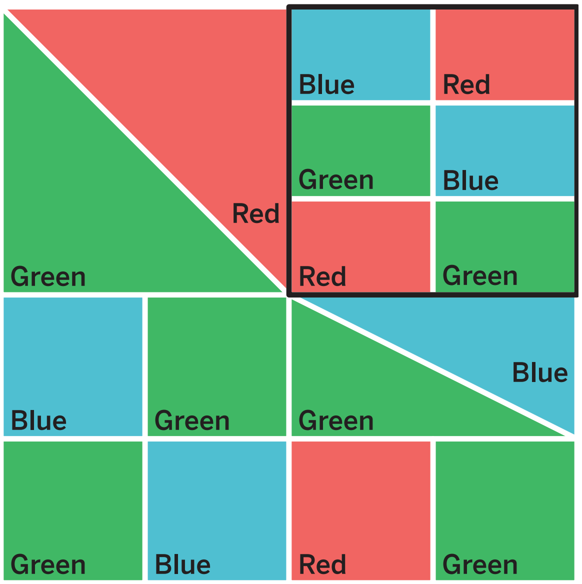
Practice

The square is composed of different shapes in three different colors.



1. Identify *at least one* way in which you see $\frac{1}{2}$ represented in the square.

2. The large square can also be broken into four smaller squares. One of these smaller squares is highlighted in the image shown. What fraction of one of these sections does each color cover?

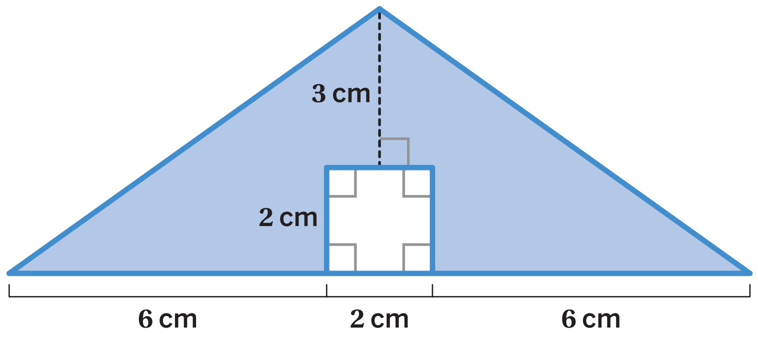


3. Han, Shawn, and Bard are considering the area covered by each color in the top right square. Han says that each color covers $\frac{1}{3}$. Shawn says each color covers $\frac{1}{6}$. Bard says each color covers $\frac{1}{12}$. Who is correct? Explain your thinking.

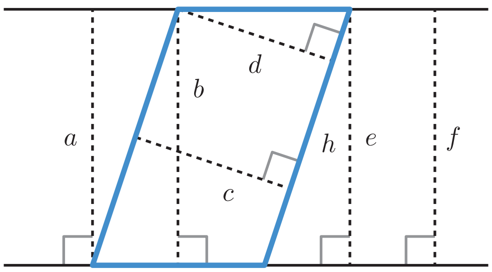
Name: ______ Date: ______ Period: ______

Practice

4. Find the area of the shaded region. Show or explain your thinking.



5. In the figure, side h is the base of the parallelogram. Select *all* the segments that could represent the height of the parallelogram.



A. Segment a
B. Segment b
C. Segment c
D. Segment d
E. Segment e
F. Segment f
G. Segment g
H. Segment h

6. A chef has a 12-lb bag of rice. Each day she uses $\frac{1}{2}$ lb of rice for various dishes served at her restaurant. Which of the following equations will help find the number of days that the bag of rice will last? Select *all* that apply.

A. $12 - \frac{1}{2} = ?$
B. $12 \cdot ? = \frac{1}{2}$
C. $? \cdot \frac{1}{2} = 12$
D. $\frac{1}{2} \cdot ? = 12$
E. $12 \div \frac{1}{2} = ?$
F. $12 \div ? = \frac{1}{2}$
G. $? \div 12 = \frac{1}{2}$
H. $? \div \frac{1}{2} = 12$

My Notes:

SUB-UNIT 1

Interpreting Division Scenarios

Welcome to Spöklik Furniture

An eeriness settles over Spöklik Furniture. Once upon a time, people came here for new couches, fine china, and silver serving spoons. Now, it's nothing but an abandoned old warehouse — full of cobwebs and unsold furniture . . . So, what was your friend Maya doing out here? She texted, asking to meet here. Now, you find her phone lying by the entrance, battery dead. You walk past the shopping carts to the store's first section: Housewares.

Clicking on your flashlight, you start your search. After a few minutes of wandering, the place starts to feel like a maze! You try to trace your steps back to the entrance, but end up going in circles. Suddenly, you hear a pair of voices:

"Martha! We can't afford *that*!"

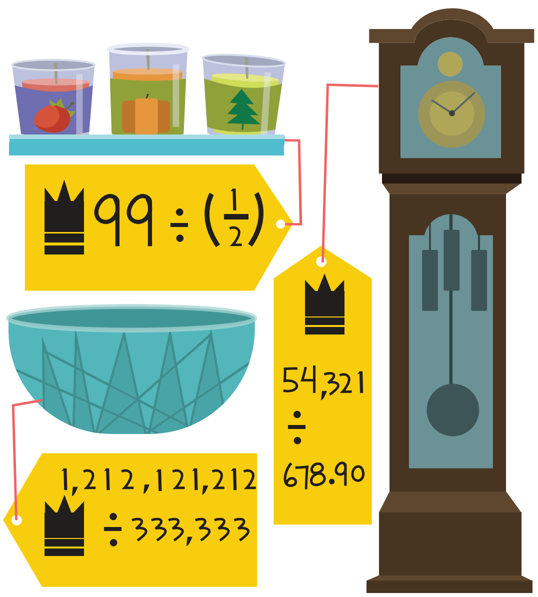
"Oh, live a little, George! You don't want the Albees calling us cheap, do you?"

Two figures appear, pushing a shopping cart. Martha cradles a crystal salad bowl in her arms. "You there!" she coos, gesturing toward you. "Be a darling and lend us a hand."

"What's the problem?" you ask.

"We're buying a house warming gift for our friends, the Albees. The trouble is, we can't decide what to get. If the gift is too cheap, it'll be a scandal. But if it's too expensive, George will have a fit. **We need an item that costs between 100 and 1,000 spök-bucks**, but these prices are *so* confusing . . ."

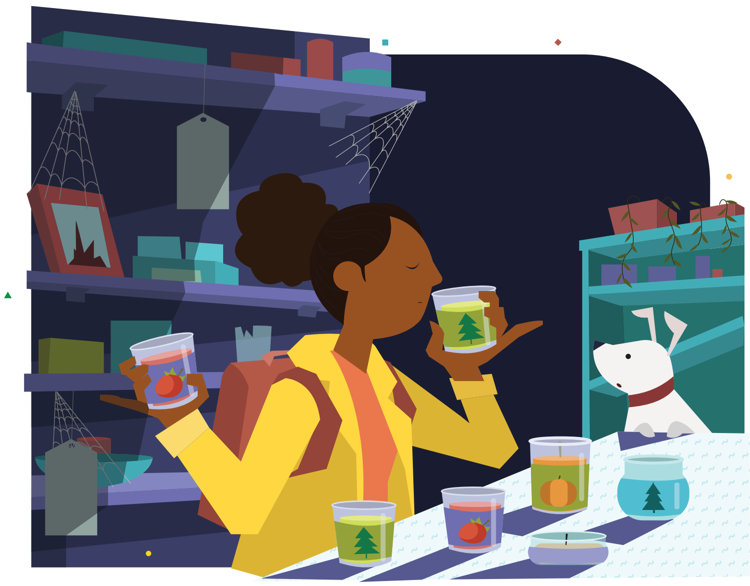
Looking at their cart, you see that, instead of showing the price, each tag is printed with a strange division problem. George and Martha look at you with imploring eyes. Maybe if you help them, they can help you get out of this place . . .



Unit 2 | Lesson 2

Relating Division and Multiplication

Let's review how division and multiplication are related.



Warm-up Fact Families

Complete each column header with a third number to form a fact family. Then write the two multiplication equations and two division equations that correspond to each fact family in the blank rows of that column.

5, 20, ______	12, 60, ______	$\frac{1}{2}$, 10, ______

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Multiplication or Division?

Some Spöklik employees are making scented jar candles. They each use melted wax to fill their jars in different ways. For each problem:

- **Choose an operation that could be used to solve the problem.**
- **Write an equation with your chosen operation, using a question mark for the unknown.**
- **Draw a diagram to help you solve for the unknown in your equation.**
- **Write the solution to the problem in a complete sentence.**

1. Mai has 4 jars, and she puts $\frac{1}{2}$ cup of melted cinnamon-toast- scented wax in each jar. How many cups of melted wax does Mai use?

Operation:

Diagram:

Equation:

Solution:

2. Priya has $\frac{1}{2}$ cup of pumpkin-frost-scented wax. She puts an equal amount of the melted wax into 4 jars. How many cups of wax are in each jar?

Operation:

Diagram:

Equation:

Solution:

3. Han has 4 cups of pine-scented wax to put into jars. If he puts $\frac{1}{2}$ cup of wax in each jar, how many jars can he fill?

Operation:

Diagram:

Equation:

Solution:

Activity 2 Multiplication and Division

For each problem:

- **Write a multiplication equation *and* a division equation.**
  Use a question mark to represent the unknown in each equation.
- **Estimate a solution to the problem.**
- **Draw a diagram to represent and to solve the problem. Explain your thinking.**
- **Write your solution in a complete sentence.**

1. Lin filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?

Multiplication equation: **Division equation:** **Estimated solution:**

Diagram:

Solution and explanation:

2. Diego had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

Multiplication equation: **Division equation:** **Estimated solution:**

Diagram:

Solution and explanation:

Are you ready for more?

Using the numbers $2\frac{2}{3}$ and 8, write all of the possible multiplication and division expressions. Then order the expressions, based on their values, from least to greatest.

STOP

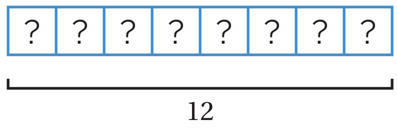
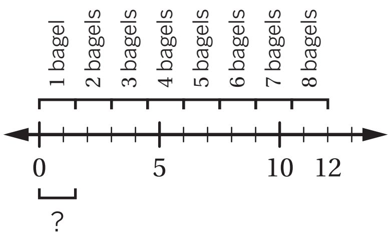
Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You revisited the relationship between the operations of multiplication and division to wrote related equations to determine the unknown values in a scenario.

For example, consider a scenario where 12 oz of cream cheese is being divided amongst 8 bagels. To determine the amount of cream cheese per bagel, you can represent the scenario using diagrams, a multiplication equation or division equation.

Tape diagram	Number line
	

Multiplication expression	Division expression
$8 \bullet ? = 12$	$12 \div 8 = ?$

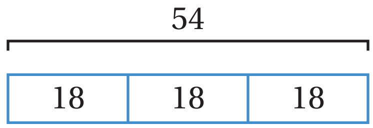
In each representation, the missing value is $1\frac{1}{2}$, which can be interpreted as "$1\frac{1}{2}$ oz of cream cheese on each of the 8 bagels for a total of 12 oz" or "12 oz of cream cheese divided evenly onto 8 bagels is$1\frac{1}{2}$ oz on each bagel."

Reflect:

Practice

Name: ________ Date: ________ Period: ________

1. Write a multiplication equation and a division equation that could be represented by the diagram shown.



2. Mai has $\$36$ to spend on movie tickets. Each ticket costs $\$4.50$.

a Write a multiplication equation and a division equation that could be used to determine how many tickets Mai can buy.

b Draw a diagram to determine the number of tickets Mai can buy.

c How many tickets can Mai buy?

3. Write a real-world problem that could be represented by the equation.

$4 \div 1\frac{1}{3} = ?$

Name: ______________________ Date: ____________ Period: ____________

4. Draw each of the indicated figures using the grids shown.

a. Draw a parallelogram that has an area of 24 square units, but is not a rectangle. Explain or show how you know the area is 24 square units.

b. Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.

5. Evaluate each expression.

a. $\frac{1}{2} \cdot \frac{2}{3}$

b. $\frac{1}{12} \cdot \frac{9}{8}$

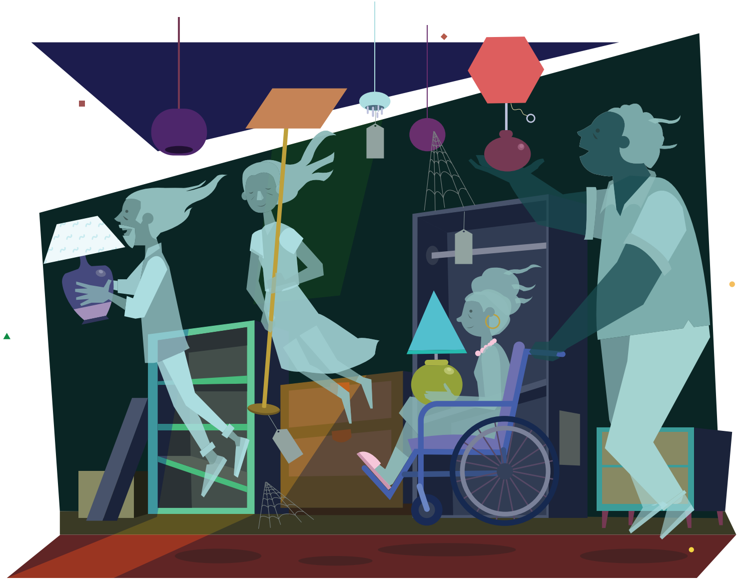
c. $\frac{1}{8} + \frac{5}{8}$

d. $\frac{5}{8} - \frac{1}{4}$

Unit 2 | Lesson 4

How Many Groups?

Let's use blocks and diagrams to think about division with fractions.



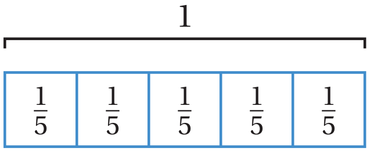
Warm-up Equal-sized Groups

Write a multiplication equation and a division equation to represent each statement or diagram.

1. Eight \$5 bills are worth \$40.

2. There are 9 thirds in 3 ones.

3.



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Name: .... Date: .... Period: ....

Activity 1 Reasoning With Pattern Blocks

Use the pattern blocks to solve the problems in Parts 1 and 2.

Part 1

If a hexagon represents 1 whole, what fractions of a whole does each of the following shapes or combinations of shapes represent? Show or explain your thinking.

1. 1 triangle

2. 1 rhombus

3. 1 trapezoid

4. 4 triangles

5. 2 hexagons and 1 rhombus

Part 2

You will be given a sheet that defines a different shape as representing 1 whole. Use the pattern blocks to determine the value represented by each shape or combination of shapes. Be prepared to explain your thinking.

Activity 2 When the Size of the Group Is a Fraction

1. A hexagon represents 1 whole. Draw a diagram by using pattern block shapes to represent each multiplication equation.

   a $3 \cdot \frac{1}{6} = \frac{1}{2}$

   b $2 \cdot \frac{3}{2} = 3$

Plan ahead: How will knowing the shape that represents 1 whole help you use an organized approach to the activity?

2. Write an equation that could be used to represent each question. Use a ? for the unknown. Then solve the equation.

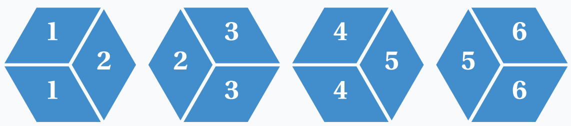
   a How many $\frac{1}{2}$s are in 4?

   b How many $\frac{2}{3}$s are in 2?

   c How many $\frac{1}{6}$s are in $1\frac{1}{2}$?

Are you ready for more?

Which of the following can be represented by these pattern blocks? Select *all* that apply.



A. How many $\frac{2}{3}$s are in 4?

B. How many 4s are in $\frac{2}{3}$?

C. $? \cdot \frac{2}{3} = 4$

D. $4 \div \frac{2}{3} = ?$

E. How many $\frac{1}{2}$s are in 4?

STOP

Name: ________________ Date: ________ Period: ________

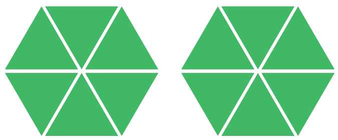
Summary

In today's lesson . . .

You looked at equal-sized groups problems where the size of each group was known, but was not a whole number. One such problem is, "How many $\frac{1}{6}$s are in 2?" To answer this question, you can write division and multiplication equations, such as $2 \div \frac{1}{6} = ?$ and $? \cdot \frac{1}{6} = 2$. You can also represent such problems by using pattern blocks, such as the ones shown.

If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$ because 6 triangles make 1 hexagon, which also represents $\frac{6}{6}$. So, answering the question, "How many $\frac{1}{6}$s are in 2?," is the same as answering, "How many triangles make two hexagons?"

The value 12 makes both equations true: $2 \div \frac{1}{6} = 12$ and $12 \cdot \frac{1}{6} = 2$. In terms of equal-sized groups, the size of each group is $\frac{1}{6}$ and 12 of them make 2.



This also works when the total is not a whole number, such as with $\frac{3}{2} \div \frac{1}{6} = ?$, which is the same as asking, "How many triangles make three trapezoids?" Either way, the answer is 9.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. In the figure, the hexagon represents 1 whole. Determine how many $\frac{1}{3}$s are in $1\frac{2}{3}$. Show or explain your thinking.

2. A shopper buys cat food in 3-lb bags. Her cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

   a. Draw a diagram to represent the scenario. Label your diagram.

   b. Write a multiplication or division equation to represent the scenario.

   c. Determine how many weeks one bag lasts. Explain your thinking.

Name: .... Date: .... Period: ....

3. Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?

A. How many 3s are in $\frac{1}{8}$?

B. What is 3 groups of $\frac{1}{8}$?

C. How many $\frac{1}{8}$s are in 3?

D. What is $\frac{1}{8}$ of 3?

4. Noah and his friends are going to an amusement park. The total cost for 8 admission tickets is $\$100$, and each person pays the same admission price. Noah brought $\$13$. Did he bring enough money for an admission ticket to the park? Show or explain your thinking.

5. Write a division expression with a quotient that is:

a Greater than $8 \div 0.001$.

b Less than $8 \div 0.001$.

c Between $8 \div 0.001$ and $8 \div \frac{1}{10}$.

6. Write a division equation that could be represented by this tape diagram.

7	7	7	7	7	7	7	7

Unit 2 | Lesson 4

Using Diagrams to Determine the Number of Groups

Let's use blocks and diagrams to understand more about division with fractions.

Warm-up Reasoning With Fraction Strips

You will be given a set of fraction strips. Write the fraction or whole number that answers each question. Be prepared to share your thinking.

1. How many $\frac{1}{2}$s are in 2?

2. How many $\frac{1}{5}$s are in 3?

3. How many $\frac{1}{8}$s are in $1\frac{1}{4}$?

4. $1 \div \frac{2}{6} = ?$

5. $6 \div \frac{2}{5} = ?$

6. $4 \div \frac{2}{10} = ?$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 More Reasoning With Pattern Blocks

Use the pattern blocks to complete Problems 1–4.

1. How many rhombuses are in one trapezoid? Show or explain your thinking.

2. If a triangle represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

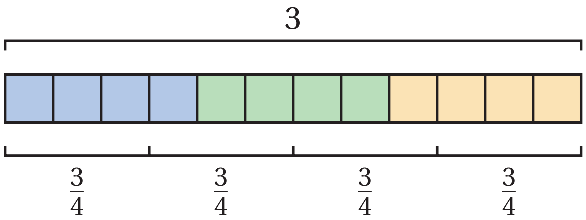
3. If a rhombus represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

4. If a trapezoid represents 1 whole, write at least two equations that could be used to determine how many rhombuses are in a trapezoid. Use a ? to represent the unknown.

Activity 2 Representing Fractional-Sized Groups

Fractional units have been used as a way of calculating with more and more precision for centuries. In the 12th century, Indian mathematician Bhāskara II used these ideas to calculate the "instantaneous" motion of a planet – how fast it traveled over very short intervals of time, which he determined could be calculated to at least 1 truti $\left(\frac{1}{33{,}750}\text{ seconds}\right)$. Knowing how many of those intervals fit into another time period, the planet's next location could be determined.

Consider how this diagram could represent time intervals of $\frac{3}{4}$ second in 3 seconds.



Part 1

Write a different story problem (that does not need to be related to planets or time) to represent the diagram, and include a question to find an unknown in the problem. Write a multiplication equation and a division equation that represents the diagram and that could be used to answer the question from your story problem. You may use a ? to represent the unknown in the equations.

Featured Mathematician

Bhāskara II

Indian mathematician and astronomer Bhāskara II, also known as "Bhāskara, the teacher" (c. 1114–1185 CE), was a major contributor to early Indian mathematics. His work covered a variety of topics, including differential calculus – the study of rates of change between two quantities, and particularly at a single instant. He did this work nearly 500 years before many European mathematicians commonly credited for similar discoveries were even born.

Name: ______ Date: ______ Period: ______

Activity 2 Representing Fractional-Sized Groups (continued)

Part 2

For Problems 1–3, write a multiplication equation and a division equation that can be used to answer the question. Then draw a tape diagram to represent the problem, and determine the solution.

1. How many $\frac{3}{4}$s are in 1?

   Multiplication equation:

   Division equation:

   Tape diagram:

   Solution:

2. How many $\frac{2}{3}$s are in 3?

   Multiplication equation:

   Division equation:

   Tape diagram:

   Solution:

3. How many $\frac{3}{2}$s are in $\frac{9}{2}$?

   Multiplication equation:

   Division equation:

   Tape diagram:

   Solution:

STOP

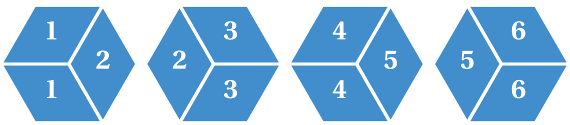
Summary

In today's lesson . . .

You continued to use representations to solve equal-sized groups problems where the size of each group was not a whole number. For example, suppose one batch of biscuits requires $\frac{2}{3}$ of a cup of flour and you want to know, "How many batches can be made with 4 cups of flour?" The size of each group is $\frac{2}{3}$ and you want to know how many groups are needed to make 4. This can be represented by the equations $4 \div \frac{2}{3} = ?$ and $? \bullet \frac{2}{3} = 4$.

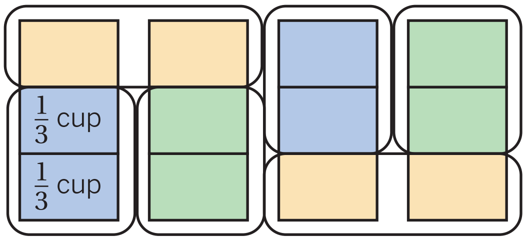
With pattern blocks, there is no single shape that represents $\frac{2}{3}$. However, because 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$, and your groups of $\frac{2}{3}$ can be represented by two rhombuses.

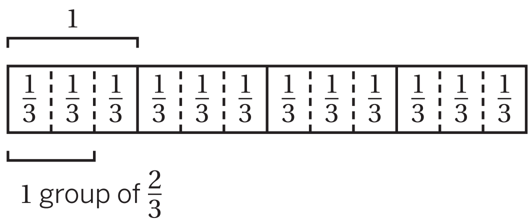
You can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4, and that means $4 \div \frac{2}{3} = 6$.



Unfortunately, pattern blocks are limited to fractions with certain denominators. But there are plenty of other kinds of diagrams that can also help you reason about equal-sized groups involving fractions, such as equipartitioned rectangles, fraction strips, and tape diagrams.

Each of these diagrams shows $4 \div \frac{2}{3} = 6$ in different ways.





Reflect:

Name: .... Date: .... Period: ....

Practice

1. The expression $3 \div \frac{1}{4}$ can be used to represent the problem "How many groups of $\frac{1}{4}$ are in 3?" Draw a tape diagram to represent this problem. Then determine the solution.

2. Describe how to draw a tape diagram to represent and solve the equation $3 \div \frac{1}{5} = ?$ You do not have to actually draw a diagram, but you may if it helps with your thinking and explanation.

Practice

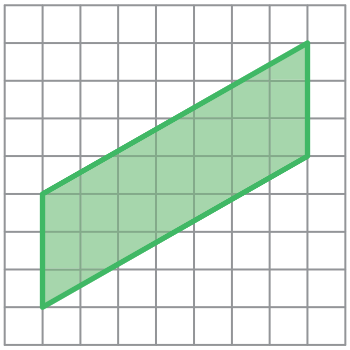
Name: ______________ Date: ______________ Period: ______________

3. Consider the problem: "How many $\frac{1}{2}$ days are there in 1 week?"

   a. Write either a multiplication equation or a division equation to represent the problem.

   b. Draw a tape diagram to represent your equation and then solve the problem.

4. Determine the area of the parallelogram. Show your thinking.



5. Evaluate each expression.

   a. $\frac{1}{2} \cdot \frac{3}{4}$

   b. $1\frac{2}{3} \cdot \frac{3}{4}$

Unit 2 | Lesson 5

Dividing With Common Denominators

Let's think about dividing things into groups using common denominators.

Warm-up Estimating a Fraction of a Number

Write a multiplication expression that could be used to solve each problem. Then use your expression to *estimate* the solution.

1. What is $\frac{1}{3}$ of 7?

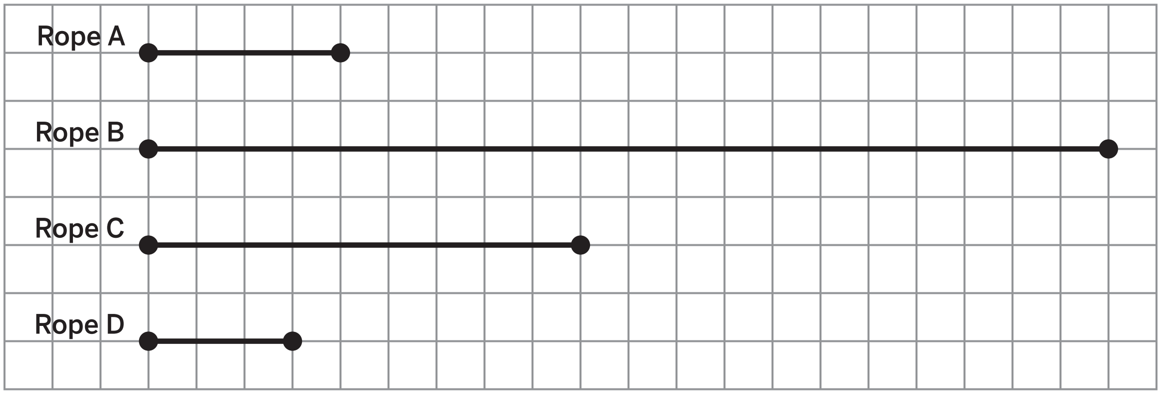
2. What is $\frac{4}{5}$ of $9\frac{2}{3}$?

3. What is $2\frac{4}{7}$ of $10\frac{1}{9}$?

Log in to Amplify Math to complete this lesson online.

Activity 1 Fractions of Ropes

These segments represent four different lengths of rope.



For each problem, write a multiplication equation and a division equation that can be used to complete the sentences comparing the lengths of the two ropes.

1. Each grid square has a length of 1 unit.

a Rope B is .... times as long as rope A.

Equations:

b Rope C is .... times as long as rope A.

Equations:

c Rope D is .... times as long as rope A.

Equations:

2. Each square represents $\frac{1}{3}$ unit.

a Rope B is .... times as long as rope A.

Equations:

b Rope C is .... times as long as rope A.

Equations:

c Rope D is .... times as long as rope A.

Equations:

Name: ______ Date: ______ Period: ______

Activity 2 Fractional Batches of Mashed Potatoes

One batch of mashed potatoes uses $4\frac{1}{2}$ lb of potatoes. A chef made different-sized batches on different days. The table shows the amounts of potatoes she used each day.

Tuesday	Wednesday	Thursday	Friday
12 lb	$7\frac{1}{2}$ lb	$6\frac{3}{4}$ lb	$1\frac{2}{3}$ lb

Three Reads: To make sense of this information, you will read this text three times. Your teacher will instruct you on what to focus for each read.

1. Write a division equation and draw a tape diagram for each day. Use both to determine how many batches of mashed potatoes she made.

a) Tuesday

b) Wednesday

c) Thursday

d) Friday

Activity 2 Fractional Batches of Mashed Potatoes (continued)

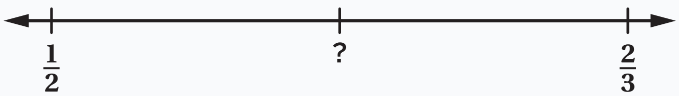
After several complaints about cold and dried-out mashed potatoes, the chef has decided to start making fresh single *servings* for every order, each using $\frac{1}{2}$ lb of potatoes. But at the end of the next week, she only has $\frac{1}{3}$ lb of potatoes left.

2. Write a division equation using common denominators and draw a diagram to represent how much of a serving she can make.

3. Imagine that she had $\frac{2}{3}$ lb of potatoes left instead. Write a division equation using common denominators to represent the situation, and then determine how much of a serving she can make.

Are you ready for more?

Determine the missing value. Show or explain your thinking.



STOP

Name: .... Date: .... Period: ....

Summary

In today's lesson . . .

You saw that common denominators can be used to help you evaluate quotients involving fractions. This is helpful for both determining the quotients and also estimating and interpreting the results. For example, $\frac{1}{4} \div \frac{1}{3}$ is asking, "How many groups of $\frac{1}{3}$ make $\frac{1}{4}$?" Rewriting that as $\frac{3}{12} \div \frac{4}{12}$ gives you the insight that the answer will be less than one whole group! And so the question could be rephrased several ways:

- "How much of a group of $\frac{1}{3}$ makes $\frac{1}{4}$?"
- "How many times as large as $\frac{1}{3}$ is $\frac{1}{4}$?"
- "What fraction of $\frac{1}{3}$ is $\frac{1}{4}$?"

The common denominator, 12 in this example, also represents a unit fraction that divides evenly into both given fractions. So, from the example, there are 4 twelfths in one third, and there are 3 twelfths in one fourth. And this means both quantities are being measured using the same units (twelfths), which means the expression can be interpreted using whole numbers. However, many times 4 ones goes into 3 ones is the same as the number of times 4 twelfths goes into 3 twelfths, or even 4 fifty-ninths goes into 3 fifty-ninths. The quotient is always equal to $3 \div 4$, which can be written as a fraction, $\frac{3}{4}$.

In general, once you have common denominators, the quotient of $\frac{a}{c} \div \frac{b}{c}$ is equal to the numerator of the dividend divided by the numerator of the divisor, $a \div b$. And better yet, that can always be written as the fraction $\frac{a}{b}$.

Reflect:

Practice

Name: .... Date: .... Period: ....

1. A recipe calls for $\frac{1}{2}$ lb of flour for 1 batch. How many batches can be made with each of these amounts? Show or explain your thinking.

   a. 1 lb

   b. $\frac{3}{4}$ lb

   c. $\frac{1}{4}$ lb

2. Whiskers, the cat, weighs $2\frac{2}{3}$ kg. Piglio weighs 4 kg. For each problem, write a multiplication and division equation and decide whether the solution is greater than 1 or less than 1. Then determine the solution.

   a. How many times as heavy as Piglio is Whiskers?

   b. How many times as heavy as Whiskers is Piglio?

3. Draw a tape diagram to represent the problem: What fraction of $2\frac{1}{2}$ is $\frac{4}{5}$? Then determine the solution.

Name: .... Date: .... Period: ....

Practice

4. How many groups of $\frac{3}{4}$ are in each of these quantities? Show or explain your thinking.

a $\frac{11}{4}$

b $6\frac{1}{2}$

5. Which problem can be represented by the equation $4 \div \frac{2}{7} = ?$

A. What are 4 groups of $\frac{2}{7}$?

B. How many $\frac{2}{7}$s are in 4?

C. What is $\frac{2}{7}$ of 4?

D. How many 4s are in $\frac{2}{7}$?

6. Use the numbers 20, 5, and 4 to write a division expression with a quotient that is:

a Greater than 1.

b Less than 1.

c Close to 1, but not equal to 1.

My Notes:

Spöklik Furniture: The Showroom

"Excellent!," Martha says. "The Albees will *adore* this. You've been so wonderful — much more helpful than that girl with the ghastly yellow jacket!" *Yellow jacket . . .? Maya's jacket is yellow*!

"We passed her in the showroom," George says. "Just through there." He points to a set of doors. *Odd. That wasn't there before* . . . You race for the door.

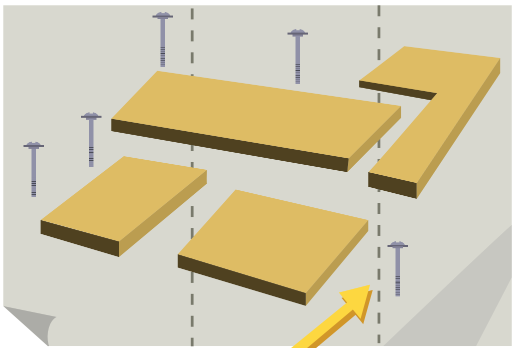
Spöklik's showroom is as grand as it is confusing: a massive warehouse with model living rooms, bedrooms, kitchens, and bathrooms. Once, shoppers wandered through this maze of displays, looking for furniture or decoration ideas. Now, the place has a strange, lonely quality. Lamplight from the model rooms spills out into the wide aisles as you step quietly, searching for Maya.

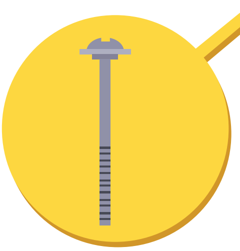
Suddenly, there is a crash! Turning the corner, you find a ghostly woman in coveralls sitting in the aisle, surrounded by tools and furniture parts. Her name tag reads: SAMIRA, SPÖKLIK TEAM MEMBER.

'PICKLES!', she swears, scattering a pile of dowels with a kick. She looks up, noticing you. "Sorry. Didn't mean to scare you. It's just that I've been building this thing for a *lifetime* now . . ." She gestures to a loose pile of boards and you realize you have no idea what it's supposed to be — a bookshelf? A dresser? A bed? "Maybe if we put our heads together, I can finally get this thing built. All I need is a Number 42 serrated flange bolt with a struntprat stem."

Samira points to the picture on her instructions. "See? The stem needs to be *this* long. The problem is I can't make heads or tails of this number!"

How long is the bolt Samira needs?



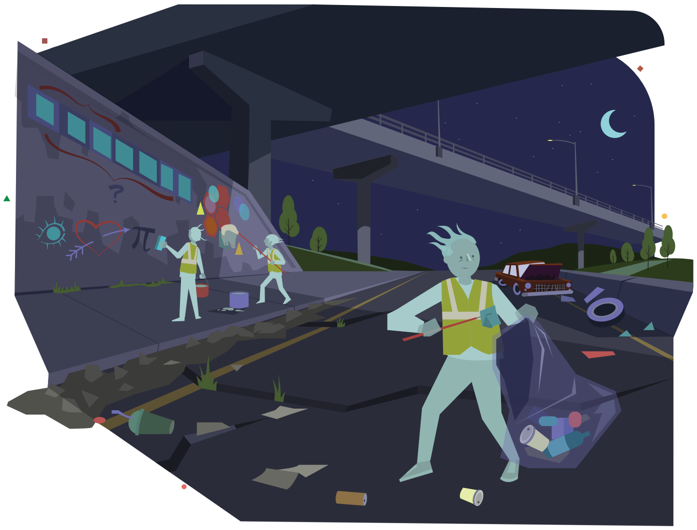


$$3+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{2}}}\ \text{mm}$$

Unit 2 | Lesson 6

How Much in Each Group?

Let's practice dividing fractions in real-world scenarios.



Warm-up Relating Dividends, Divisors, and Quotients

1. Without calculating, decide whether each quotient is greater than 1 or less than 1. Be prepared to explain your thinking.

   a. $\frac{1}{2} \div \frac{1}{4}$

   b. $1 \div \frac{3}{4}$

   c. $\frac{2}{3} \div \frac{7}{8}$

   d. $2\frac{7}{8} \div 2\frac{3}{5}$

Log in to Amplify Math to complete this lesson online.

Name: ______ Date: ______ Period: ______

Activity 1 An Adopted Highway

Three sixth grade classes adopted different sections of a highway to keep clean. Represent each scenario with a tape diagram, a division equation, and a multiplication equation. Then determine how long of a highway section each class adopted.

1. Priya's class adopted two equal-sized sections of the highway. The combined length of the two sections is $\frac{3}{4}$ mile long. How long is each section?

   a **Tape diagram:**

   b **Division and multiplication equations:**

   c **Solution:**

2. Han's class adopted one section of the highway. The length of $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long. How long is the whole section?

   a **Tape diagram:**

   b **Division and multiplication equations:**

   c **Solution:**

3. Lin's class adopted some equal-sized sections of the highway. The combined length of $1\frac{1}{2}$ sections is $\frac{3}{4}$ mile long. How long is each section?

   a **Tape diagram:**

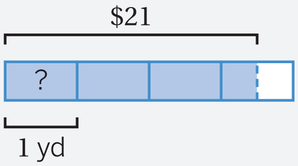
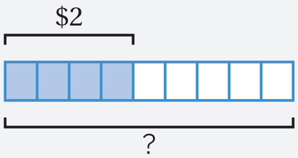
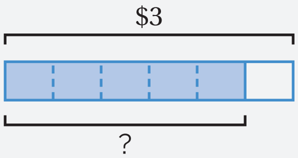
   b **Division and multiplication equations:**

   c **Solution:**

Activity 2 Reupholstering a Chair

George and Martha loved the style of one particular decorative chair that was on clearance at Spöklik. However, the chair was on clearance for a reason — the fabric was hideous! They decided to buy the chair anyway and pay for the premium upgrade of custom reupholstering. Because they could not agree on a color or pattern, they decided to only consider the cost.

To help understand the odd ways that prices and measurements are done at Spöklik, George and Martha started to create this table, but have not completed it. Use the given information to complete the table.

Scenario	Estimated solution	Tape diagram	Division equation	Solution
$3\frac{1}{2}$ yd of purple-and-white-striped fabric costs \$21. How much does 1 yd cost?	\$7		$21 \div 3\frac{1}{2} = ?$	1 yd costs \$6.
$\frac{4}{9}$ yd of leopard-print fabric costs \$2. How much does 1 yd cost?			$2 \div \frac{4}{9} = ?$	
$1\frac{1}{5}$ yd of blue-velvet fabric costs \$3. How much does 1 yd cost?				1 yd costs \$2.50.
$\frac{3}{8}$ yd of fabric is enough to reupholster $\frac{1}{3}$ of the chair. How much fabric is needed for the entire chair?			$\frac{3}{8} \div \frac{1}{3} = ?$	$1\frac{1}{8}$ yd of fabric is needed to reupholster the entire chair.

STOP

Name: ________________ Date: ________ Period: ________

Summary

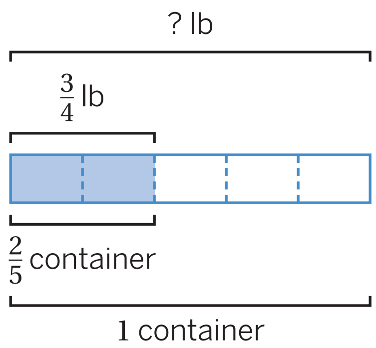
In today's lesson . . .

You looked at equal-sized groups problems where the total and number of groups were known, but the size of each group was unknown. These types of scenarios are probably familiar from working on fair sharing problems previously, and they can also still be represented by both division and multiplication equations. Depending on the context, it is important to think about which quantity represents the groups.

For example, consider this scenario: $\frac{3}{4}$ lb of rice fills $\frac{2}{5}$ of a container. There are two possible groups: pounds or containers. So, there are two different questions you could ask, and each requires different equations and diagrams.

How many pounds in 1 container?

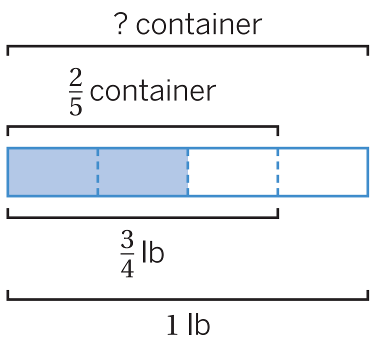
$\frac{2}{5} \cdot ? = \frac{3}{4}$ $\qquad$ $\frac{3}{4} \div \frac{2}{5} = ?$



Because $\frac{2}{5}$ of a container can be filled with $\frac{3}{4}$ lb of rice, then $\frac{1}{5}$ of a container could be filled with half of that, or $\frac{3}{8}$ lb. This means the amount in one whole container is equal to $5 \cdot \frac{3}{8}$, or $\frac{15}{8}$ lb.

How many containers for 1 lb?

$\frac{3}{4} \cdot ? = \frac{2}{5}$ $\qquad$ $\frac{2}{5} \div \frac{3}{4} = ?$



Because $\frac{3}{4}$ lb can fill $\frac{2}{5}$ of a container, then $\frac{1}{4}$ lb could fill $\frac{1}{3}$ of $\frac{2}{5}$, or $\frac{2}{15}$ of a container. This means one whole pound could fill $4 \cdot \frac{2}{5}$, or $\frac{8}{15}$ of a container.

Reflect:

Practice

Name: ____________________ Date: __________ Period: __________

1. Tyler painted $\frac{9}{2}$ yd^2 of wall area with 3 gallons of paint. How many gallons of paint does it take to paint each square yard of wall?

2. After walking $\frac{1}{4}$ mile from home, Han is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?

   a. Write a multiplication equation and a division equation to represent this situation.

   b. Complete the diagram to represent the scenario.

   c. Determine the solution.

3. After it rained for $\frac{3}{4}$ of an hour, a rain gauge is $\frac{2}{5}$ of the way filled. If it continues to rain at that same rate for 15 more minutes, what fraction of the rain gauge will be filled?

   a. To help solve this problem, Diego wrote the equation $\frac{3}{4} \div \frac{2}{5} = ?$ Explain why this equation does *not* represent the problem.

Vadym Zaitsev/Shutterstock.com

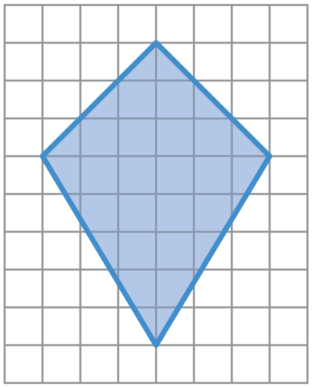
   b. Write a multiplication equation and a division equation that represents this problem.

   c. Use your equations to solve the problem.

Name: ________ Date: ________ Period: ________

Practice

4. Determine the area of this quadrilateral. Show or explain your strategy.



5. Consider the division equation: $\frac{4}{5} \div \frac{2}{3} = ?$

a. Write a story (including a question) that would represent the equation.

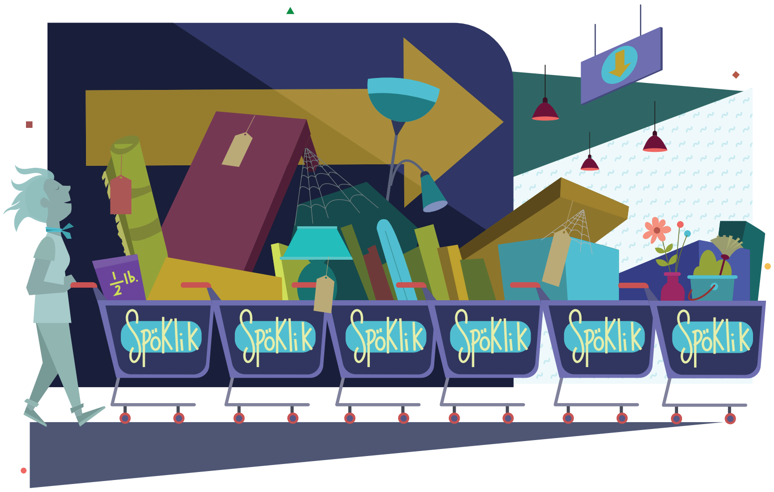
b. Determine the answer to the question.

6. For each statement, write a corresponding multiplication expression or division expression. Then evaluate your expression.

a. The product of three fourths and one third.

b. The quotient of 5 and $\frac{1}{3}$.

Dividing by Unit and Non-Unit Fractions



Let's look for patterns when we divide by fractions.

Warm-up Dividing by a Whole Number

You and your partner will each work with the problems in one column. For each of your problems, write an equation that could be used to solve the problem with the indicated operation. Then draw a diagram to show your thinking and determine the solution.

Partner A

How many 3s are in 12?

Division equation:

How many 4s are in 12?

Division equation:

Partner B

How much is 12 groups of $\frac{1}{3}$?

Multiplication equation:

How much is 12 groups of $\frac{1}{4}$?

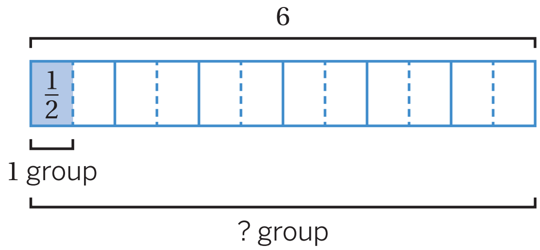
Multiplication equation:

Name: .... Date: .... Period: ....

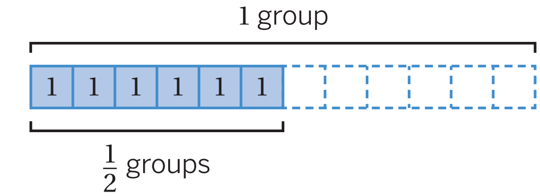
Activity 1 Dividing Whole Numbers by Fractions

Elena and Diego are trying to determine the quotient of the expression $6 \div \frac{1}{2}$.

Elena thought of the problem as asking, "How many $\frac{1}{2}$s are in 6?" and she drew this tape diagram.



Diego thought of the problem as asking, "If there are 6 in $\frac{1}{2}$ of a group, how much is in 1 group?" and he drew this tape diagram.



Choose *either* Elena's method or Diego's method and use it for Problems 1–4.

1. For each division expression, draw a diagram to represent the quotient. Then determine the value of the quotient.

a $6 \div \frac{1}{3}$

Value of the expression: ....

b $6 \div \frac{2}{3}$

Value of the expression: ....

Activity 1 Dividing Whole Numbers by Fractions (continued)

2. For each division expression, draw the diagram to represent the quotient. Then determine the value of the quotient.

   a. $6 \div \frac{1}{4}$

   Value of the expression: ....

   b. $6 \div \frac{3}{4}$

   Value of the expression: ....

3. Examine the expressions, diagrams, and quotients from Problems 1 and 2. Look for any patterns. Describe what you notice.

4. Choose the correct word for each blank to make a true statement.

numerator	denominator

Dividing a number by a fraction is the same as multiplying by the .... of the fraction and dividing by the .... of the fraction.

Name: .... Date: .... Period:

Activity 2 Dividing Fractions by Fractions

Choose one of the methods for dividing whole numbers by fractions from Activity 1 to determine whether it still works when the dividend is *not* a whole number. For each division expression, draw a diagram and then determine the quotient. Be prepared to explain your thinking.

Plan ahead: How can you use diagrams to help more clearly communicate your thinking?

1. $\frac{8}{9} \div \frac{2}{3}$

2. $\frac{7}{8} \div \frac{5}{4}$

STOP

Summary

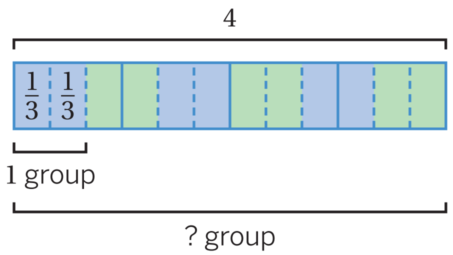
In today's lesson . . .

You compared the similarities and differences in the process of solving problems such as, "How many $\frac{1}{3}$s are in 4?" and "What is $4 \div \frac{1}{3}$?"

You can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4. In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b, which is the ***reciprocal*** of $\frac{1}{b}$.

How can you reason about $4 \div \frac{2}{3}$? You already know that there are $(4 \cdot 3)$, or 12, groups of $\frac{1}{3}$s in 4. To determine how many $\frac{2}{3}$s are in 4, you need to place every 2 of the $\frac{1}{3}$s into a group. Doing this results in half as many groups, which is 6 groups. In other words:



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

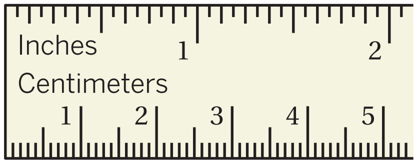
$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

Reflect:

Name: .... Date: .... Period: ....

1. A centimeter ruler is shown.



a Use the ruler to determine the quotients of $1 \div \frac{1}{10}$ and $4 \div \frac{1}{10}$.

b What calculation did you use each time?

c Use this pattern to determine $18 \div \frac{1}{10}$.

d Explain how you could determine the quotients of $4 \div \frac{2}{10}$ and $4 \div \frac{8}{10}$.

2. Determine each quotient.

a $5 \div \frac{1}{10}$

b $5 \div \frac{3}{10}$

c $5 \div \frac{9}{10}$

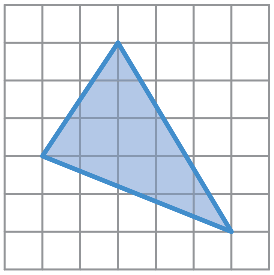
3. Use the equation $2\frac{1}{2} \div \frac{1}{8} = 20$ to determine $2\frac{1}{2} \div \frac{5}{8}$. Show or explain your thinking.

Name: .... Date: .... Period: ....

4. A box contains $1\frac{3}{4}$ lb of pancake mix. Jada used $\frac{7}{8}$ lb for a recipe. What fraction of the pancake mix in the box did she use?

   Show or explain your thinking.

5. Determine the area of the triangle. Show your thinking.



6. Determine the fractional value of each division problem.

   a. $5 \div 9$

   b. $5 \div 2$

   c. $2 \div 10$

Name: ________________ Date: ________ Period: ________

Unit 2 | Lesson 8

Using an Algorithm to Divide Fractions

Let's divide fractions by using the rule we learned.

Warm-up Multiplying Fractions

Evaluate each expression. Be prepared to explain your thinking.

1. $\frac{2}{3} \cdot 27$

2. $\frac{1}{2} \cdot \frac{2}{3}$

3. $\frac{2}{9} \cdot \frac{3}{5}$

4. $\frac{27}{100} \cdot \frac{200}{9}$

5. $1\frac{3}{4} \cdot \frac{5}{7}$

Log in to Amplify Math to complete this lesson online.

Activity 1 Exploring the Fraction Division Algorithm

Consider this statement from Lesson 7: "In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a, or multiplying the number first by b and then by $\frac{1}{a}$.

1. Select *all* of the expressions that represent the same value as $n \div \frac{a}{b}$.

A. $n \bullet \frac{a}{b}$

B. $n \bullet a \div b$

C. $n \bullet b \div a$

D. $n \div a \bullet b$

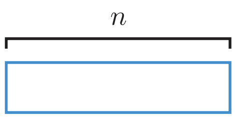
E. $n \div b \bullet a$

F. $n \bullet \frac{b}{a}$

G. $n \div \frac{b}{a}$

H. $\frac{n}{\frac{a}{b}}$

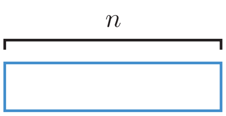
2. This tape diagram represents a number n.



a Explain how you would use the tape diagram to show $n \div \frac{a}{b}$.

Stronger and Clearer:
You'll meet with 2–3 partners to give and receive feedback on your responses to Problem 2. Use this feedback to improve your response.

b Use the tape diagram to show $n \div \frac{3}{4}$.



Name: ______ Date: ______ Period: ______

Activity 1 Exploring the Fraction Division Algorithm (continued)

3. Select *all* of the expressions that *always* have the same value as $\frac{c}{d} \div \frac{a}{b}$.

A. $\frac{c}{d} \cdot \frac{a}{b}$

B. $\frac{c}{d} \cdot a \div b$

C. $\frac{c}{d} \cdot b \div a$

D. $\frac{c}{d} \div a \cdot b$

E. $\frac{c}{d} \div b \cdot a$

F. $\frac{c}{d} \cdot \frac{b}{a}$

G. $\frac{c}{d} \div \frac{b}{a}$

H. $\frac{a}{b} \cdot \frac{c}{d}$

I. $\frac{a}{b} \div \frac{c}{d}$

J. $\frac{b}{a} \div \frac{d}{c}$

K. $c \div d \cdot a \div b$

L. $c \div d \cdot b \div a$

M. $c \div d \div a \cdot b$

N. $\dfrac{\frac{c}{d}}{\frac{a}{b}}$

Activity 2 Practice Dividing Fractions

Recall from Lesson 4 that Bhāskara II used fractions in developing notions of differential calculus in 12th century India. Nearly 800 years later, those two topics are still actively being used by mathematicians, such as Ron Buckmire. Buckmire has been working on a model for predicting what fraction of a film's total earnings (or "gross") comes after its opening weekend. Given two films, how could you compare which will perform better? By dividing their fractions, of course.

Here are several division expressions that could represent any two quantities you might want to compare. Evaluate each expression by dividing the fractions.

1. $\frac{2}{5} \div \frac{1}{3}$

2. $1\frac{1}{4} \div \frac{2}{5}$

3. $\frac{9}{10} \div 1\frac{2}{9}$

4. $\dfrac{4}{\frac{2}{3}}$

5. $\dfrac{\frac{2}{3}}{4}$

6. $\dfrac{\frac{3}{5}}{\frac{4}{7}}$

Featured Mathematician

Ron Buckmire

Born in Grenada, Ron Buckmire is a Professor of Mathematics and the Associate Dean for Curricular Affairs and Director of the Core Program at Occidental College. He is also a co-founder of the Barbara Jordan/Bayard Rustin Coalition, a civil rights organization. Buckmire's mathematical research focuses on numerical analysis and applied mathematics, including mathematical modeling. For example, he applied ordinary differential equations to develop a model for predicting the time evolution of theatrical film grosses.

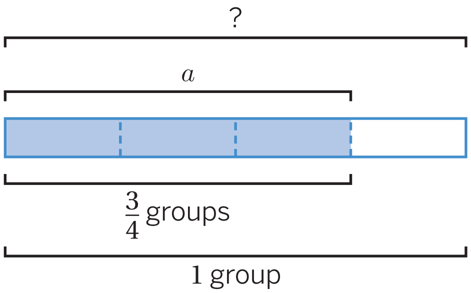
STOP

Name: ________ Date: ________ Period: ________

Summary

In today's lesson . . .

You saw that the division equation $a \div \frac{3}{4} = ?$ is equivalent to the multiplication equation $\frac{3}{4} \cdot ? = a$, so you can think of it as meaning "$\frac{3}{4}$ of what number is a?" and represent it with a diagram as shown. The length of the entire diagram represents the unknown number.



If $\frac{3}{4}$ of a number is a, then you can first divide a by 3 to determine $\frac{1}{4}$ of the number. Then you multiply the result by 4 to determine the number.

The steps above can be written as: $a \div 3 \cdot 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so you can also find the number by using the expression: $a \cdot \frac{1}{3} \cdot 4$.

In other words, because $a \div 3 \cdot 4 = a \cdot \frac{1}{3} \cdot 4$ and $a \cdot \frac{1}{3} \cdot 4 = a \cdot \frac{4}{3}$, you have the result: $a \div \frac{3}{4} = a \cdot \frac{4}{3}$. In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the ***reciprocal*** of the fraction.

Reflect:

Practice

Name: ________________ Date: ________ Period: ________

1. Select *all* the statements that provide the correct steps for evaluating the expression $\frac{14}{15} \div \frac{7}{5}$.

   A. Multiply $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

   B. Divide $\frac{14}{15}$ by 5, and then multiply by $\frac{1}{7}$.

   C. Multiply $\frac{14}{15}$ by 7, and then multiply by $\frac{1}{5}$.

   D. Multiply $\frac{14}{15}$ by 5, and then divide by 7.

2. Claire said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$ because $\frac{4}{3} \bullet 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$. Explain why Clare's quotient and reasoning are incorrect. Determine the correct quotient.

3. Determine the value of each of the following.

   a. $\frac{8}{9} \div 4$

   b. $\frac{3}{4} \div \frac{1}{2}$

   c. $\frac{\frac{9}{2}}{\frac{3}{8}}$

   d. $3\frac{1}{3} \div \frac{2}{9}$

Name: ______ Date: ______ Period: ______

Practice

4. Consider the problem: After charging for $\frac{1}{3}$ of an hour, a phone is at $\frac{2}{5}$ of its full power. How long will it take the phone to charge completely? Decide whether each equation can represent the situation. Write *yes* or *no*.

a. $\frac{1}{3} \cdot ? = \frac{2}{5}$

b. $\frac{1}{3} \div \frac{2}{5} = ?$

c. $\frac{2}{5} \div \frac{1}{3} = ?$

d. $\frac{2}{5} \cdot ? = \frac{1}{3}$

5. Elena and Noah are each filling a bucket with water. Noah's bucket is $\frac{2}{5}$ full and the water weighs $2\frac{1}{2}$ lb. How much does Elena's water weigh if her bucket is full and her bucket is identical to Noah's?

a. Write a multiplication and a division equation to represent the scenario.

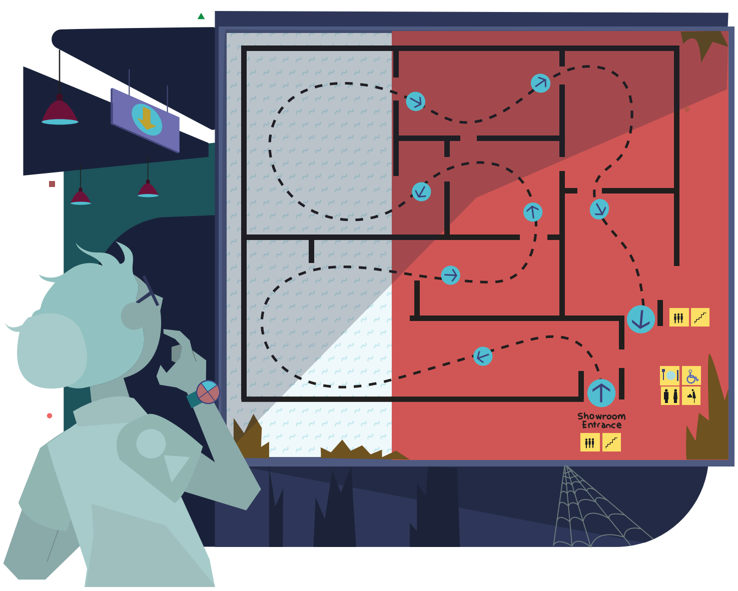
b. Draw a diagram to show the relationship between the quantities and determine the answer.

6. Without calculating, determine how the expressions $98 \cdot 25$ and $(100 \cdot 25) - (2 \cdot 25)$ are related. Explain your thinking.

Unit 2 | Lesson 18 ×

Related Quotients

Let's solve division problems by using related quotients.



Warm-up Number Talk

Mentally determine the product: $19 \cdot 14$. Be prepared to explain your thinking.

Log in to Amplify Math to complete this lesson online.

Name: .... Date: .... Period: ....

Activity 1 Related Division Expressions

1. Write a division expression that can help answer each of these questions. Then show your work for evaluating each expression to determine a solution, and write your solution as a complete sentence.

a. How many groups of $\frac{3}{8}$ are in 6?

Expression: **Solution:**

b. How many groups of $\frac{3}{4}$ are in 12?

Expression: **Solution:**

c. How many groups of $\frac{3}{2}$ are in 24?

Expression: **Solution:**

d. How many groups of 3 are in 48?

Expression: **Solution:**

e. How many groups of $\frac{1}{4}$ are in 4?

Expression: **Solution:**

Activity 1 Related Division Expressions (continued)

2. Consider your solutions for Problems 1a–1e.

a How are your quotients related?

b How are your expressions related? Explain your thinking.

3. Write a division expression that would result in the same quotient as $3 \div \frac{3}{16}$, where both the dividend and divisor are *whole numbers*. Explain your thinking.

Name: ________ Date: ________ Period: ________

Activity 2 Using Related Quotients

A Spöklik security guard is investigating a strange noise reported in the Furniture Department. Looking at the store map, she estimates that she has covered about $\frac{2}{3}$ of the department in roughly $\frac{3}{4}$ of an hour. Will she complete her investigation of the entire department in one hour?

1. Write an expression to represent the problem and then evaluate your expression.

Expression: **Evaluation:**

2. Explain how your expression and its quotient help you to determine whether the security guard will finish investigating the entire Furniture Department in one hour.

3. Write a related division expression that results in the same quotient, but with *either* the dividend *or* the divisor written as a unit fraction. Explain how you created your related expression.

4. Explain what the dividend and divisor mean in context. Use a strategy other than the algorithm to show how the quotient gives you the same information as Problem 1.

STOP

Summary

In today's lesson . . .

You saw that you can solve division problems by using related expressions. When you multiply or divide both the dividend and the divisor by the same number, the result is a related expression with the same quotient.

Related division expressions that feature two whole numbers or a unit fraction are particularly useful. For example, to evaluate $20 \div \frac{4}{3}$, you could multiply both the dividend and divisor by 3 to make the related expression $60 \div 4$, which equals 15. You could also divide both the dividend and the divisor by 4 to make the related expression $5 \div \frac{1}{3}$.

You can evaluate any of these quotients by using the algorithm or ratio thinking.

Algorithm	Ratio Thinking
$20 \div \frac{4}{3}$ $20 \div \frac{4}{3} = \frac{20}{1} \div \frac{4}{3}$ $= \frac{20}{1} \cdot \frac{3}{4}$ $= \frac{60}{4}$ or 15	$5 \div \frac{1}{3}$ • There are 3 groups of $\frac{1}{3}$ in 1. • There are 6 groups in 2, 9 groups in 3, 12 groups in 4, and 15 groups in 5. • There are 15 groups of $\frac{1}{3}$ in 5.

All such related division expressions will result in the same quotient of 15.

Reflect:

Name: ________________ Date: ________ Period: ________

Practice

1. Write a division expression to represent each problem. Then evaluate your expression to solve each problem. Explain your thinking.

a How many groups of $\frac{2}{5}$ are in 4?

b How many groups of $\frac{4}{5}$ are in 8?

c How many groups of $\frac{6}{5}$ are in 12?

2. What fraction of $3\frac{1}{2}$ is $\frac{3}{4}$?

a Write a division expression to represent the problem. Then evaluate your expression.

Expression: **Evaluate:**

b Write a related division expression that results in the same quotient, but where both the dividend and divisor are *whole numbers*. Explain your thinking.

3. Bard is walking their dog on a path that is $\frac{4}{5}$ mile and has already walked $\frac{2}{3}$ mile. What fraction of the path has Bard already walked? Show or explain your thinking by using a method other than the algorithm.

Practice

Name: .... Date: .... Period: ....

4. A group of friends are equally sharing $2\frac{1}{2}$ lb of berries.

a If each friend receives $\frac{5}{4}$ lb of berries, how many friends are sharing the berries?

b If 5 friends are sharing the berries, how many pounds of berries does each friend receive?

5. Order quotients from least to greatest. Show your thinking.

$3.75 \div 10,\ 37.5 \div 1000,\ 3.75 \div 0.1,\ 37.5 \div 0.1,\ 0.375 \div 0.1$

Least Greatest

6. Kiran and Clare are comparing the numbers 1,000 and 10. Kiran says that 1,000 is 100 times as large as 10. Clare says that 10 is $\frac{1}{100}$ times as large as 1,000. Who is correct? Explain your thinking.

Spöklik Furniture: Checking Out

"Is this what you were looking for?" a voice asks. You look up and your heart almost leaps out of your chest. You see a girl in a yellow jacket: Maya!

"Ah, perfect!" Samira says. Samira takes the bolt from Maya's outstretched hand. As Samira continues working, Maya throws her arms around you. Her dog, Penny, barks happily behind her.

"You two have been super helpful," Samira says. "Thanks for everything!"

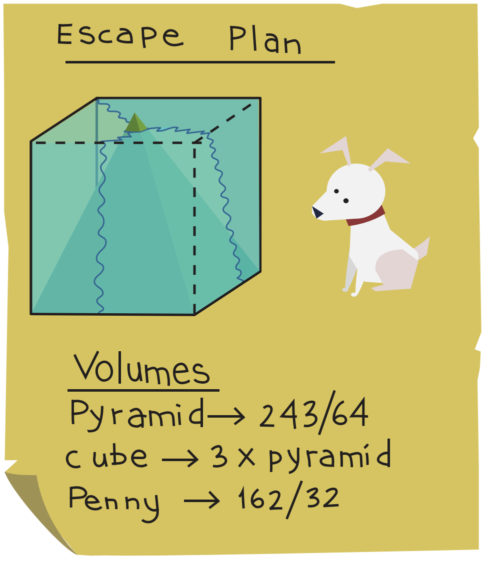
"No problem," Maya says, "We'd better be going now!" Maya takes you by your wrist and, together, you make your way out through a stairwell.

"I'm so glad I found you. There's an exit through the checkout section, but the guards won't let me through with Penny. They're convinced she belongs in the store!"

At the bottom of the stairs, you step out into a massive room. Spectral shoppers hover in line, waiting to check out. Beyond the row of registers are the exit doors, watched over by the security guards Maya warned you about.

Suddenly, you spot a shopper toward the back of one of the lines. This shopper seems distracted by a set of tea towels. In their cart, you see a large cardboard box. Suddenly, an idea occurs to you: You can sneak Penny through by hiding her in the box!

You, Maya, and Penny quietly creep behind the cart. Opening the box, you find a strange sight: a jade statue shaped like a pyramid, surrounded by 3 squishy foam packets. **"We'll have to get rid of some of these packets for Penny to fit," Maya whispers. "But how many?"**



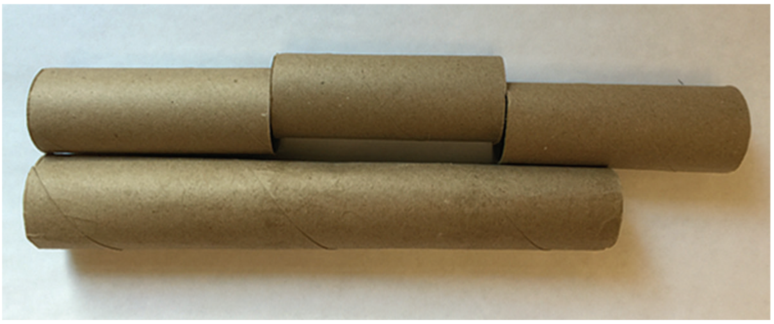
Unit 2 | Lesson 10

Fractional Lengths

Let's solve problems about fractional lengths.

Warm-up Comparing Paper Rolls

The image shows bath tissue rolls and a paper towel roll. Let b represent the length of a bath tissue roll and let p represent the length of a paper towel roll. Write a multiplication equation and a division equation that represents the length of one roll in terms of the other.



Log in to Amplify Math to complete this lesson online.

Name: ________ Date: ________ Period: ________

Activity 1 How Many Times as Tall or as Long?

1. A security guard at Spöklik's self-checkout likes to hide behind a potted plant while monitoring the area for theft. The plant is 4 ft tall, and the security guard is $5\frac{2}{3}$ ft tall. Write a division expression that could be used to answer each question. Then evaluate your expressions and determine the solutions.

   a. How many times as tall as the plant is the guard?

   Expression: **Evaluate:**

   Solution:

   b. What fraction of the guard's height is the plant's height?

   Expression: **Evaluate:**

   Solution:

Activity 1 How Many Times as Tall or as Long? (continued)

2. The security guard works $9\frac{1}{2}$-hour long shifts. At one point during a shift, the guard looked at the clock and realized it had been $3\frac{3}{4}$ hours since the shift started.

   a. Without calculating, determine if the guard has worked *at least* half of the shift. Explain your thinking.

   b. Calculate exactly how much of the shift the guard has worked. Show your thinking.

   c. Is your answer to part b reasonable based on your answer to part a? Explain your thinking.

Are you ready for more?

An envelope has a perimeter of $18\frac{1}{3}$ in., and its width is $\frac{2}{3}$ as long as its length. What is the area of the envelope?

Name: ______________________ Date: __________ Period: __________

Activity 2 Info Gap: Decorating Notebooks

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card:*	If your teacher gives you the *data card:*
1. Silently read your card. Think about what information you need to be able to answer the question.	**1.** Silently read your card.
2. Ask your partner for the specific information that you need.	**2.** Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information. If your partner asks for information that is not on the card, tell them you do not have that information.
3. Explain how you are using the information to solve the problem. Continue to ask questions until you have enough information to solve the problem.	**3.** Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Share the *problem card* and solve the problem independently.	**4.** Read the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.	**5.** Share the *data card* and discuss your reasoning.

STOP

Summary

In today's lesson . . .

You saw that division can help to solve comparison problems in which you determine how many times as large one quantity is compared to another.

For example, consider the lengths of two songs from a sixth grade chorus concert. The first song is $1\frac{1}{2}$ minutes long and the second song is $3\frac{3}{4}$ minutes long. You can compare the lengths of the two songs by asking either of two different questions, as shown in the table.

How many times as long as the first song is the second song?	What fraction of the second song is the first song?
$? \cdot 1\frac{1}{2} = 3\frac{3}{4}$ $3\frac{3}{4} \div 1\frac{1}{2} = ?$ $= \frac{15}{4} \div \frac{3}{2}$ $= \frac{15}{4} \cdot \frac{2}{3}$ $= \frac{30}{12}$ or $\frac{5}{2}$ or $2\frac{1}{2}$ The second song is $2\frac{1}{2}$ times as long as the first song.	$? \cdot 3\frac{3}{4} = 1\frac{1}{2}$ $1\frac{1}{2} \div 3\frac{3}{4} = ?$ $= \frac{3}{2} \div \frac{15}{4}$ $= \frac{6}{4} \div \frac{15}{4}$ $= \frac{6}{15}$ or $\frac{2}{5}$ The first song is $\frac{2}{5}$ as long as the second song.

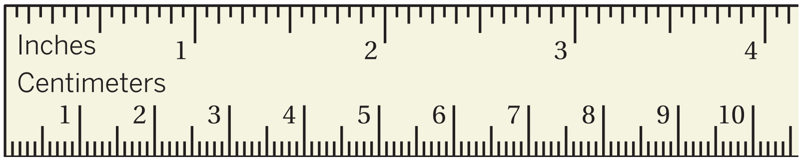
Both questions can be represented by using different pairs of multiplication and division equations, and both can be answered by using any of the strategies you have seen for division.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. One in. is about the same length as $2\frac{27}{50}$ cm.



a About how many centimeters long is 3 in.? Show your thinking.

b Using this approximation, what fraction of 1 in. is 1 cm? Show your thinking.

c What question can be answered by determining $10 \div 2\frac{27}{50}$ in this context?

2. A zookeeper is $6\frac{1}{4}$ ft tall. A young giraffe is $9\frac{3}{8}$ ft tall.

a What fraction of the giraffe's height is the zookeeper? Show your thinking.

b How many times as tall is the giraffe than the zookeeper? Show your thinking.

3. A rectangular bathroom floor is covered with square tiles that have side lengths of $1\frac{1}{2}$ ft. The length of the bathroom floor is $10\frac{1}{2}$ ft and the width is $6\frac{1}{2}$ ft. How many tiles does it take to cover the:

a Length of the floor?

b Width of the floor?

Practice

Name: .... Date: .... Period: ....

4. $\frac{3}{4}$ cup of oatmeal has $\frac{1}{10}$ of the recommended daily value of iron. What fraction of the daily recommended value of iron is in 1 cup of oatmeal?

a. Write a multiplication and a division equation to represent the scenario.

b. Determine the solution to the problem. Show your thinking.

5. Clare says, "There are $2\frac{1}{2}$ groups of $\frac{4}{5}$ in 2." Do you agree with her? Show your thinking.

6. A rectangle has a length of 4 in. and an area of 14 in^2.

a. What is the width of the rectangle? Show or explain your thinking.

b. Write an equation that represents the area of the rectangle using the length of the width you calculated in part a, that shows the Commutative Property of Multiplication.

Name: ______ Date: ______ Period: ______

Unit 2 | Lesson 11

Area With Fractional Side Lengths

Let's explore the area of rectangles and triangles with fractional side lengths.

Warm-up Area Match Up

Figures A–D are rectangles with different areas, but all of their shaded regions have the *same* area.

Figure A

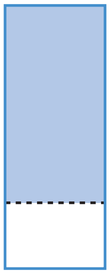


Figure B

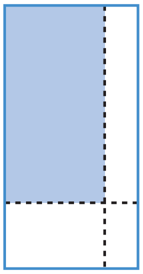


Figure C

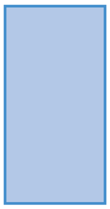
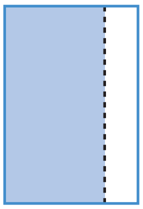


Figure D



1. Each of these expressions represents the entire area of one of the figures. Match each expression to the correct figure letter. Be prepared to explain your thinking.

   $2 \cdot 4$ ______ $2\frac{1}{2} \cdot 4$ ______

   $2 \cdot 4\frac{3}{4}$ ______ $2\frac{1}{2} \cdot 4\frac{3}{4}$ ______

2. Use the rectangle whose area is $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$ is $11\frac{7}{8}$.

Log in to Amplify Math to complete this lesson online.

Activity 1 How Many Would it Take?

Noah would like to cover a rectangular tray by using rectangular tiles with no gaps or overlaps. The tray has a width of $11\frac{1}{4}$ in. and an area of $50\frac{5}{8}$ in^2.

1. Let ℓ represent the length of the tray. Write an equation that represents the scenario and determine the length of the tray in inches. Show your thinking.

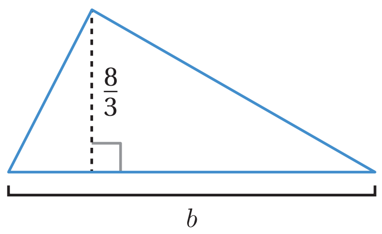
2. If each tile measures $\frac{3}{4}$ in. by $\frac{9}{16}$ in., how many tiles would Noah need to cover the tray completely? Would he need to cut or break any of the tiles? If so, what is the least number he would have to cut? Draw a diagram to show your thinking.

Name: ______________________ Date: __________ Period: __________

Activity 2 Areas of Triangles With Fractional Lengths

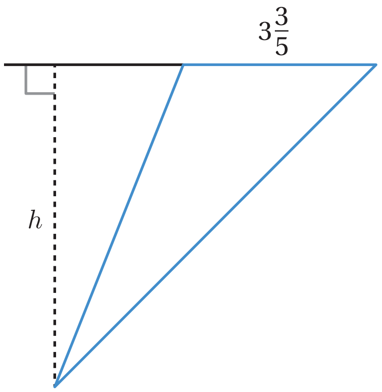
1. The area of Triangle A is 8 square units. What is the missing length b? Show your thinking.

Triangle A



2. Shawn said the missing length h of Triangle B could be determined by solving the equation $3\frac{3}{5} = \frac{1}{2} \cdot 10\frac{4}{5} \cdot h$. Do you agree or disagree? If you agree, use Shawn's equation to solve for h. If you disagree, write an equation that would solve for h and then solve your equation.

Triangle B



Area $= 10\frac{4}{5}$ square units

STOP

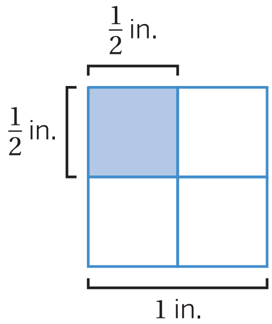
Summary

In today's lesson . . .

You applied the area formulas for parallelograms and triangles you learned in a previous unit to determine missing values when the measurements of a rectangle or triangle included fractional side lengths.

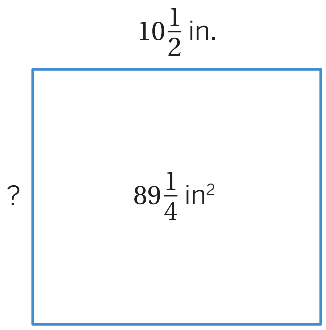
Recall that rectangles and squares are special examples of parallelograms, and for a rectangle with side lengths a units and b units, its area is equal to $a \cdot b$ square units.

This diagram shows how the formula applies to a square with a fractional side length of $\frac{1}{2}$ in. Its area is equal to the product $\frac{1}{2} \cdot \frac{1}{2}$, which means its area is $\frac{1}{4}$ in^2.



As with whole numbers, you can also use these area formulas to determine an unknown length. If you know the *area* and *one side length* of a rectangle, then you can divide to determine the *other* side length.

For example, the equation $10\frac{1}{2} \cdot ? = 89\frac{1}{4}$ shows the relationship between the area and the given side length of this rectangle. To determine the missing side length, you can divide: $89\frac{1}{4} \div 10\frac{1}{2} = ?$



And all of this also still works for a triangle with base b and height h. When one or both of those values are fractions, the area is still equal to $\frac{1}{2} \cdot b \cdot h$.

Reflect:

Name: .... Date: .... Period: ....

Practice

1. A worker is tiling the floor of a rectangular room that is 12 ft by 15 ft. The tiles are squares with a side length of $1\frac{1}{3}$ ft. How many tiles are needed to cover the entire floor? Show or explain your thinking.

2. A television screen has a length of $16\frac{1}{2}$ in., a width of w inches, and an area of 462 in^2. Select *all* the equations that represent the relationship between the dimensions of the television.

A. $w \cdot 462 = 16\frac{1}{2}$

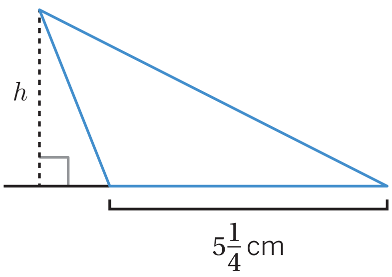
B. $16\frac{1}{2} \cdot w = 462$

C. $462 \div 16\frac{1}{2} = w$

D. $462 \div w = 16\frac{1}{2}$

E. $16\frac{1}{2} \cdot 462 = w$

3. The triangle has an area of $7\frac{7}{8}$ cm^2. What is the length of h? Explain your thinking.



Practice

Name: ________ Date: ________ Period: ________

4. A bookshelf is 42 in. long.

a If books are lined up with a width of $1\frac{1}{2}$ in., how many will fit on the bookshelf? Explain your thinking.

b A bookcase has five of these bookshelves. How many total feet of shelf space does the bookcase have? Explain your thinking.

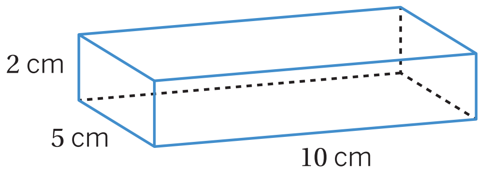
5. How many groups of $1\frac{2}{3}$ are in each of these quantities?

a $1\frac{5}{6}$

b $4\frac{1}{3}$

c $\frac{5}{6}$

6. Priya claimed that the base of this figure has an area of 50 cm^2 and the volume is 100 cm^3. Bard claimed that the the base has an area of 10 cm^2, but agreed that the volume is 100 cm^3. Can both Priya and Bard be correct? Show or explain your thinking.



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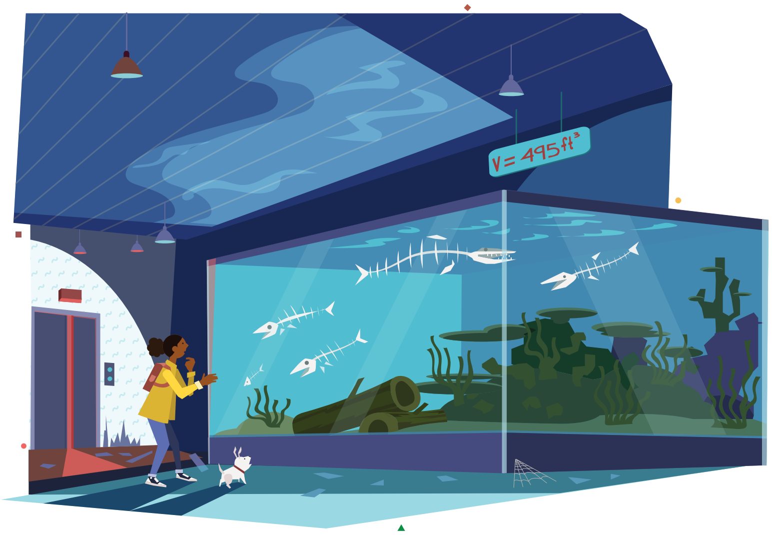
Unit 2 | Lesson $\frac{3}{5} \div$

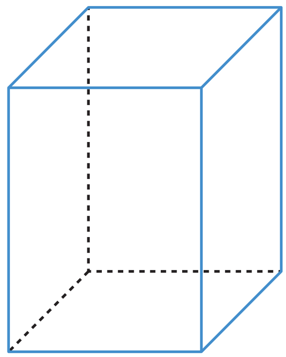


Volumes of Prisms

Let's look at the volumes of some more prisms with fractional measurements.

Warm-up Clare's Fish Tank

The figure shows Clare's fish tank. Use the figure to complete Problems 1–2. Be prepared to explain your thinking.



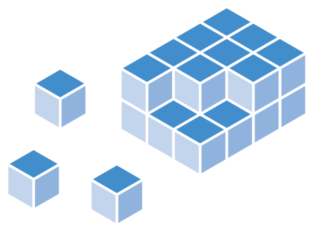
1. Assume that the length and width of the base are whole numbers. How many cubes with an edge length of 1 in. could be packed into Clare's fish tank?

2. How is the number of 1-in. cubes related to the volume of the prism?

Log in to Amplify Math to complete this lesson online.

Activity 1 Volume of Cubes and Prisms

You will be given cubes with an edge length of $\frac{1}{2}$ in. to build prisms with the lengths, widths, and heights shown in the table.



a For each prism, use the table to record how many $\frac{1}{2}$-in. cubes can be packed into the prism. Then, determine the volume of the prism.

Prism length (in.)	Prism width (in.)	Prism height (in.)	Number of $\frac{1}{2}$-in. cubes in prism	Volume of prism (in^3)
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	1	$\frac{1}{2}$		
	2	1		4
4		$\frac{3}{2}$		12

b Examine the values in the table. What is the relationship between the volume and the number of $\frac{1}{2}$-in. cubes?

c Clare's fish tank has a length of 8 in., width of 6 in. and a height of 12 in. How many $\frac{1}{2}$-in. cubes can be packed into Clare's fish tank? Explain your thinking.

Name: .... Date: .... Period: ....

Activity 2 Spöklik's Fish Tank

1. In the checkout area of Spöklik Furniture Store, there is a huge fish tank in the shape of a rectangular prism. It has a volume of 510 ft^3. The height of the tank is 10 ft and the length of $8\frac{1}{2}$ ft. What is the width of the tank? Show or explain your thinking.

For problems 2 and 3, you will work with a partner, Partner A should respond to Question A and Partner B responds to Question B. Compare and discuss your responses.

2.

Question A	Question B
How many times greater is the volume of Spöklik's fish tank than the volume of a $\frac{1}{2}$-ft cube?	How many of $\frac{1}{2}$-ft. cubes could fit in the Spöklik fish tank?

3.

Question A	Question B
How many times would Clare need to fill her fish tank in order to fill the Spöklik's fish tank?	How many of Clare's fish tanks could fit in the Spöklik fish tank?

STOP

Summary

In today's lesson . . .

You saw that, to determine the volume of a rectangular prism with fractional edge lengths, you can think of the prism as being built of cubes that have a unit fraction for their edge length.

For example, the volume of a rectangular prism with fractional edge lengths as $\frac{1}{2}$ in., 4 in., and $\frac{3}{2}$ in. can be found filling the prism using different-sized cubes.

unit cubes with 1 in. edge lengths

$\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$ of the 1 in. cubes.

The volume of the prism is 3 in^3.

cubes with $\frac{1}{2}$ in. edge lengths

$\frac{1}{2} \div \frac{1}{2} = 1$

$\frac{3}{2} \div \frac{1}{2} = 3$

$4 \div \frac{1}{2} = 8$

The volume of the prism $1 \cdot 3 \cdot 8 = 24$ of the $\frac{1}{2}$ in. cubes.

The volume of each $\frac{1}{2}$-in. cube is $\frac{1}{8}$ in^3. Therefore, the prism that can be filled with 24 of these cubes has a volume of 3 in^2; $24 \cdot \frac{1}{8} = 3$.

Recall that you applied the volume formula to determine the missing height of the Spöklik's fish tank. Dividing by fractions also helps you to calculate one edge length of a rectangular prism, given its volume and the other two edge lengths.

Reflect:

Name: ____________ Date: ____________ Period: ____________

Practice

1. A pool in the shape of a rectangular prism is being filled with water. The length of the pool is 24 ft and its width is 15 ft. When the height of the water in the pool is $1\frac{1}{3}$ ft, what is the volume of the amount of water in the pool?

2. A rectangular prism measures $2\frac{2}{5}$ in. by $3\frac{1}{5}$ in. by 2 in.

   a. Andre says, "It takes more cubes with edge length of $\frac{2}{5}$ in. than cubes with edge length of $\frac{1}{5}$ in. to pack the prism." Do you agree with Andre? Explain your thinking.

   b. How many cubes with edge length of $\frac{1}{5}$ in. would pack the prism? Explain your thinking.

   c. Show or explain how you can use your answer from part b to determine the volume of the prism in cubic inches.

3. Before refrigerators existed, some people had blocks of ice delivered to their homes. The delivery wagon contained a storage box that was a rectangular prism. Consider such a storage box that measures $7\frac{1}{2}$ ft by 6 ft by 6 ft. If the blocks of ice stored in the box were cubes with an edge length of $1\frac{1}{2}$ ft, how many blocks of ice could fit in the storage box?

   A. 270

   B. $3\frac{3}{4}$

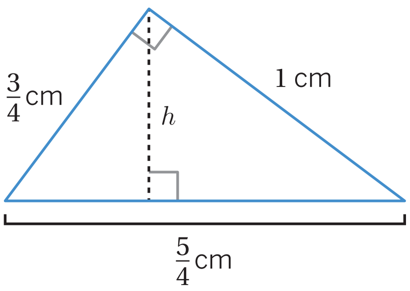
   C. 80

   D. 180

Practice

Name: ________ Date: ________ Period: ________

4. Consider the right triangle.



a. What is the area of the triangle? Show your thinking.

b. What is the height h, in centimeters, for the base that is $\frac{5}{4}$ cm long? Show your thinking.

5. Consider a bucket that contains $11\frac{2}{3}$ gallons of water and is $\frac{5}{6}$ full.

a. Write a multiplication equation *and* a division equation that could be used to determine how many gallons of water the bucket can hold when it is full.

b. How many gallons of water would be in the bucket when it is full?

6. Priya's cat weighs $5\frac{1}{2}$ lb and her dog weighs $8\frac{1}{4}$ lb. For each part, estimate a number that would complete each comparison statement. Then find an exact solution. If any of your estimates were not close to the solution, explain why that may have happened.

a. The cat is ____ as heavy as the dog.

Estimate:

Calculate:

Explain:

b. The dog is ____ lb heavier than the cat.

Estimate:

Calculate:

Explain:

Name: ______________________ Date: __________ Period: __________

Unit 2 | Lesson 13 – Capstone

Now, Where Was That Bus?

Let's solve a location mystery by using fractions.

Warm-up Hunting for Clues

After smuggling Penny past the guards — which was not too difficult because one of them seemed to be unusually preoccupied with a large fish tank — Maya ran to the elevator of the 12-floor parking garage, to catch a shuttle bus home, away from Spöklik. As she entered the elevator, a voice mysteriously called out, "Please enter your floor, followed by your row, and then your platform."

But, instead of numbers, the elevator buttons showed pictures of items from Spöklik's different departments. Maya reached into her pocket, hoping her bus ticket was still there. It was! Printed on the ticket was the following information:

There is no WAY Maya is going back into Spöklik for the information! However, there are three clues in this unit that could help. Can you help Maya by first finding the clues?

Lesson	Clue

TICKET

Location $7\frac{1}{2}$:

Each floor is divided by rows, and then divided by platforms. Visit our Housewares, Showroom and Check-out sections for more information!

(Lost values are not the responsibility of Spöklik or its employees.)

Log in to Amplify Math to complete this lesson online.

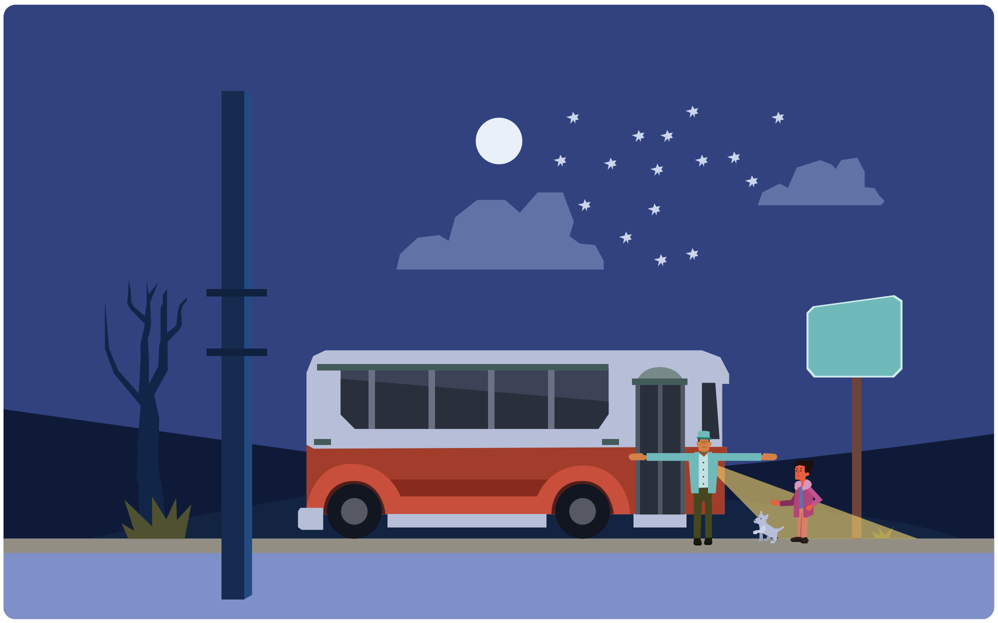
Activity 1 Determining the Right Combination

With the information from the Warm-up, determine the combination of buttons Maya needs to press in order for the elevator to take her to the correct floor, row, and platform that matches the location of the bus stop. Hint: Read the bus ticket closely.

Bus Stop Location:

Floor: .... **Row:** .... **Platform:** ....

Elevator Button Combination:



STOP

Unit Summary

Now that Maya and Penny are on their way home, it's time to set aside any fear you may have had of dividing fractions.

In your math studies, you are bound to run into concepts that might seem a little mysterious at first. But never forget that while math can come off as serious, it also has a playful side. Good mathematicians know how to have fun. They try things out, notice patterns, and even turn their problems upside down (sometimes literally!).

But like any game, there are rules. And when you know how to play with these rules, surprising things can happen. In this unit, you saw how playing with a numerator or denominator affects a quotient. You also saw how dividing and multiplying are two sides of the same coin. You even found new ways of thinking about what dividing fractions even mean: either splitting a quantity into a certain number of equal groups and asking how large they are, or splitting a quantity into groups of a certain size and asking how many there are.

These insights are possible when you are flexible and imaginative. So the next time you find yourself staring down a tough math problem, it never hurts to crack a smile and treat it like a game.

See you in Unit 3.

Practice

Name: .... Date: .... Period: ....

1. Divide. Show your thinking.

a $\frac{3}{8} \div \frac{5}{8}$

d $\frac{4}{7} \div \frac{4}{5}$

b $\frac{2}{3} \div \frac{5}{4}$

e $4 \div \frac{3}{5}$

c $\frac{1}{5} \div \frac{3}{4}$

f $3\frac{1}{2} \div \frac{2}{3}$

2. Consider the problem: How many $\frac{2}{3}$s are in 4? Represent this question using an expression, then use any strategy to answer the question.

Name: .... Date: .... Period: ....

Practice

3. Clare is using small wooden cubes with edge length of $\frac{1}{2}$ in. to build a larger cube that has edge length 4 in. How many small cubes does she need? Explain your thinking.

4. $\frac{2}{5}$ kg of soil fills $\frac{1}{3}$ of a container. Can 1 kg of soil fit in the container? Show or explain your thinking.

5. Consider the following statements.

a Shawn says that there is no number that is equal to its reciprocal. Do you agree with Shawn? Show your thinking.

b Priya says that every number has a reciprocal. Do you agree with Priya? Show your thinking.